$P_1(-2|-1)$ $P_2(2|-5)$

$y = m \cdot x + b$

I $-1 = m \cdot (-2) + b$

II $-5 = m \cdot 2 + b$

$y = (-1) \cdot x - 3$

$\mathrm{II} - \mathrm{I}$ $-4 = m \cdot 4 \quad | :4$

$m = -1$

Einsetzen z.B. in II:

$-5 = (-1) \cdot 2 + b$

$-5 = -2 + b \quad | +2$

$b = -3$

$y = m \cdot x + b$ klassische Darstellung

$f(x) = m \cdot x + b$

m Steigung

b y-Achsenabschnitt

$y = m \cdot (x - x_0) + y_0$ Punkt-Richtungs-Form

$f(x) = m \cdot (x - x_0) + y_0$

$f(x) = m \cdot (x - x_p) + y_p$

m Steigung

x_p x_0

y_p y_0

$\Big\}$ x- und y-Koordinate eines bestimmten Punktes durch den die Gerade verläuft

$P_1(-2|-1)$ $P_2(2|-5)$ Steigung: $m=-1$

$$f(x) = m\cdot(x-x_0)+y_0$$

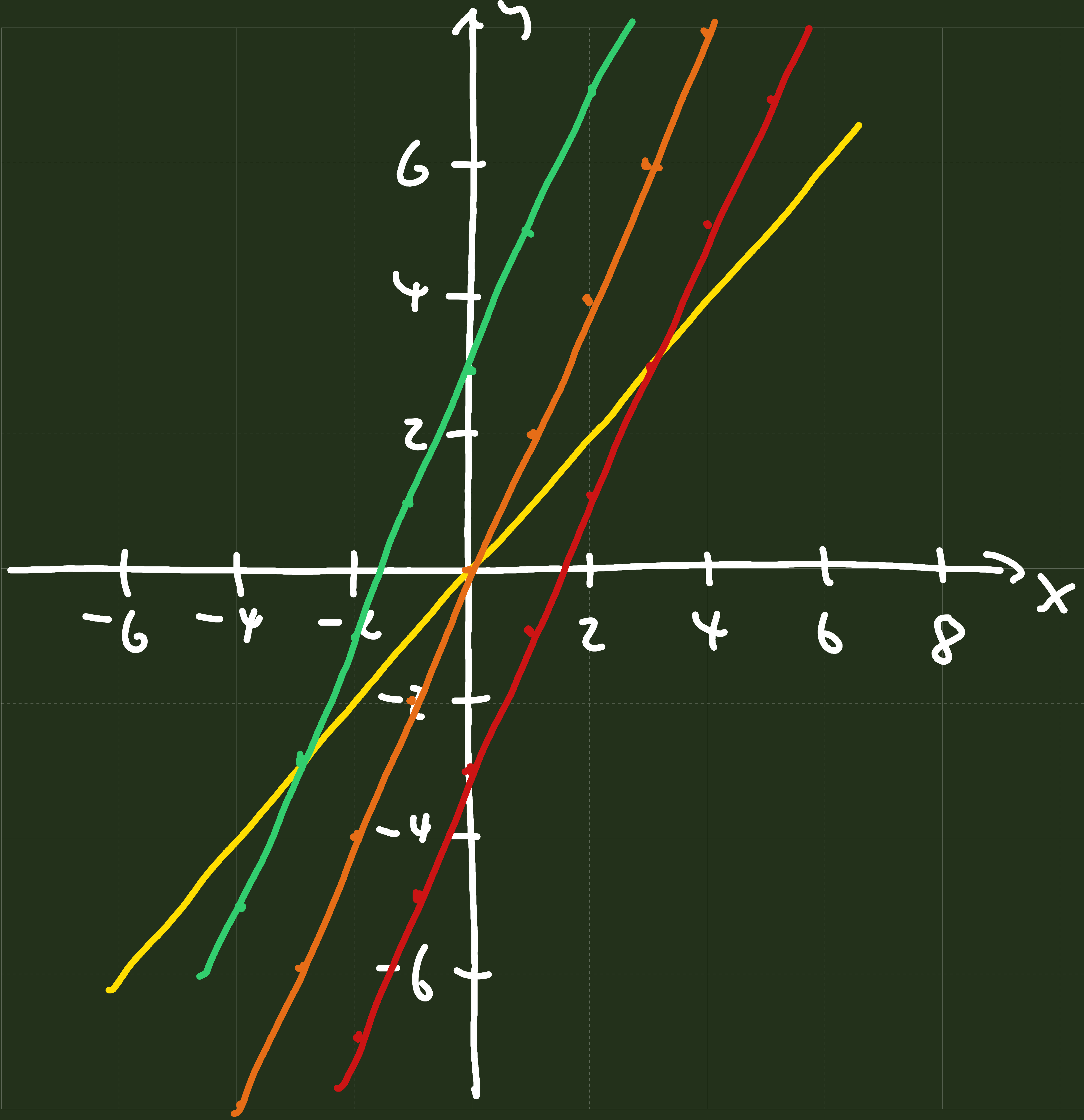
$$f(x) = -1\cdot(x-(-2))+(-1) \qquad f(x) = -1\cdot(x-2)+(-5)$$

$$= -1\cdot(x+2)-1 \qquad = -1\cdot x+2-5$$

$$= -1\cdot x-2-1 \qquad = -1\cdot x-3$$

$$= -1\cdot x-3$$

$=$ 



$y = x$ "Ursprungsgerade"

$y = 2 \cdot x$

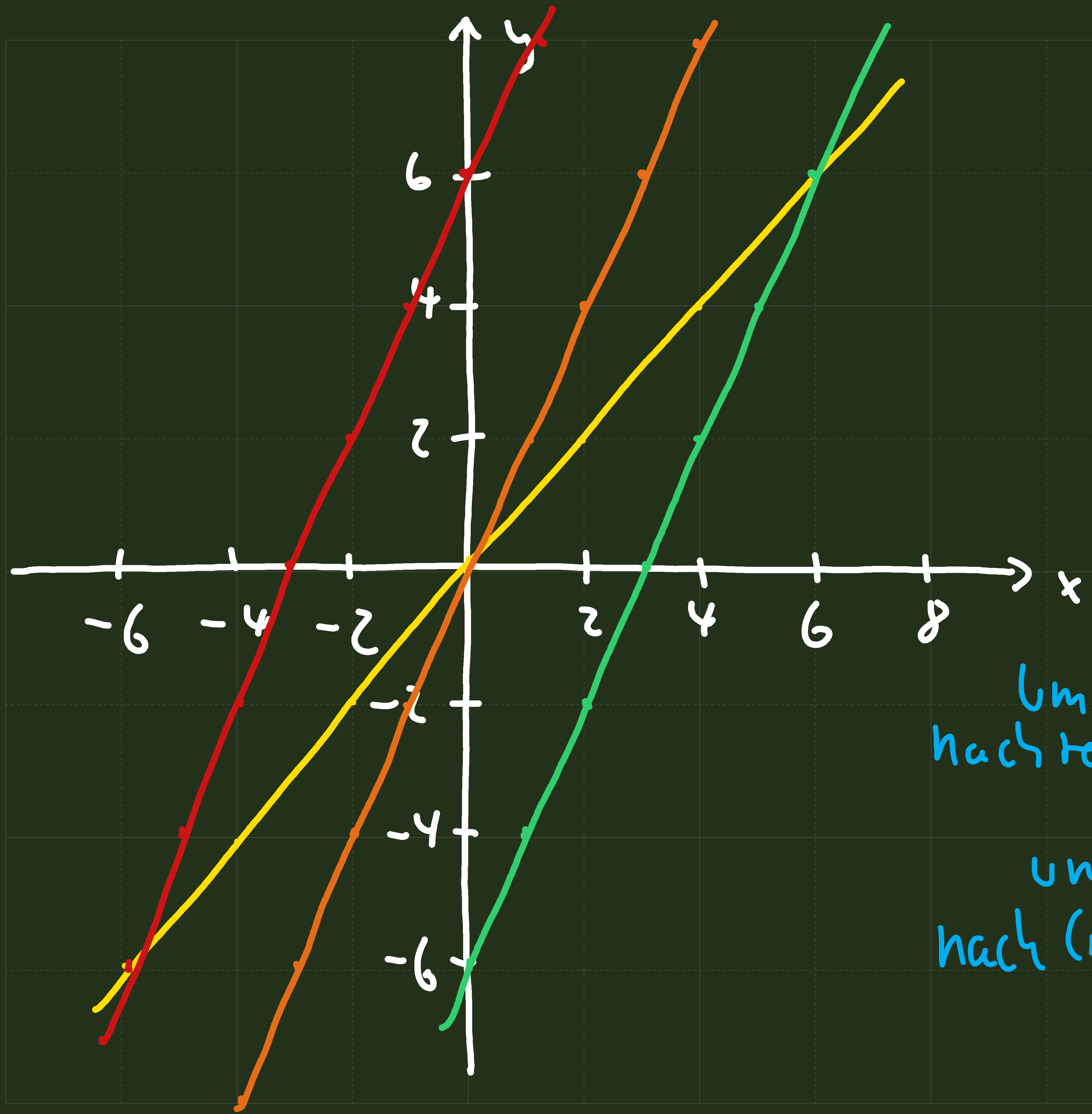
Verschieben auf der y-Achse

nach oben mit +

$y = 2 \cdot x + 3$ um 3 nach oben

nach unten mit −

$y = 2 \cdot x - 3$ um 3 nach unten



$y = x$ „Ursprungsgerade"

$y = 2 \cdot x$

Verschieben auf der X-Achse

nach rechts mit − bei X

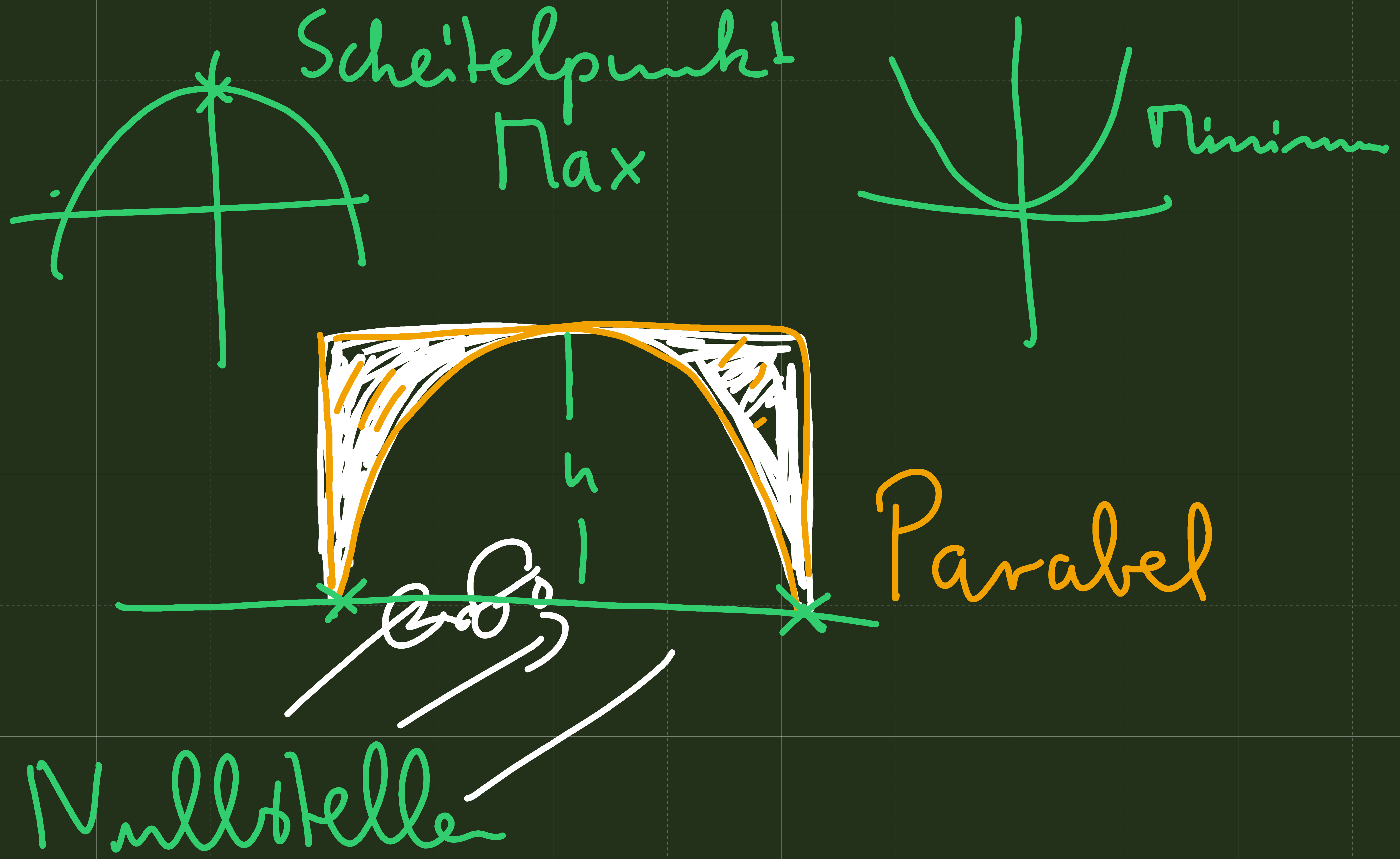
um 3 nach rechts

$$y = 2 \cdot (x - 3) = 2 \cdot x - 6$$

nach links mit + bei X

um 3 nach links

$$y = 2 \cdot (x + 3) = 2 \cdot x + 6$$


Scheitelpunkt
Max
Minimum
h
Parabel
Nullstellen

1. Nullstellen: $y = ax^2 + bx + c$

Lösung mit Hilfe der p-q-Formel

$$0 = x^2 + p \cdot x + q$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x^2+px+q=0 \quad | -q$$

$$x^2+px+\left(\frac{p}{2}\right)^2 = -q+\left(\frac{p}{2}\right)^2$$

$$x^2+2\cdot\left(\frac{p}{2}\right)x+\left(\frac{p}{2}\right)^2 = -q+\left(\frac{p}{2}\right)^2$$

$$\left(x+\frac{p}{2}\right)^2 = \left(\frac{p}{2}\right)^2-q \quad | \sqrt{}$$

$$x_{1,2}+\frac{p}{2} = \pm\sqrt{\left(\frac{p}{2}\right)^2-q} \quad | -\frac{p}{2}$$

$$x_{1,2} = -\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}$$

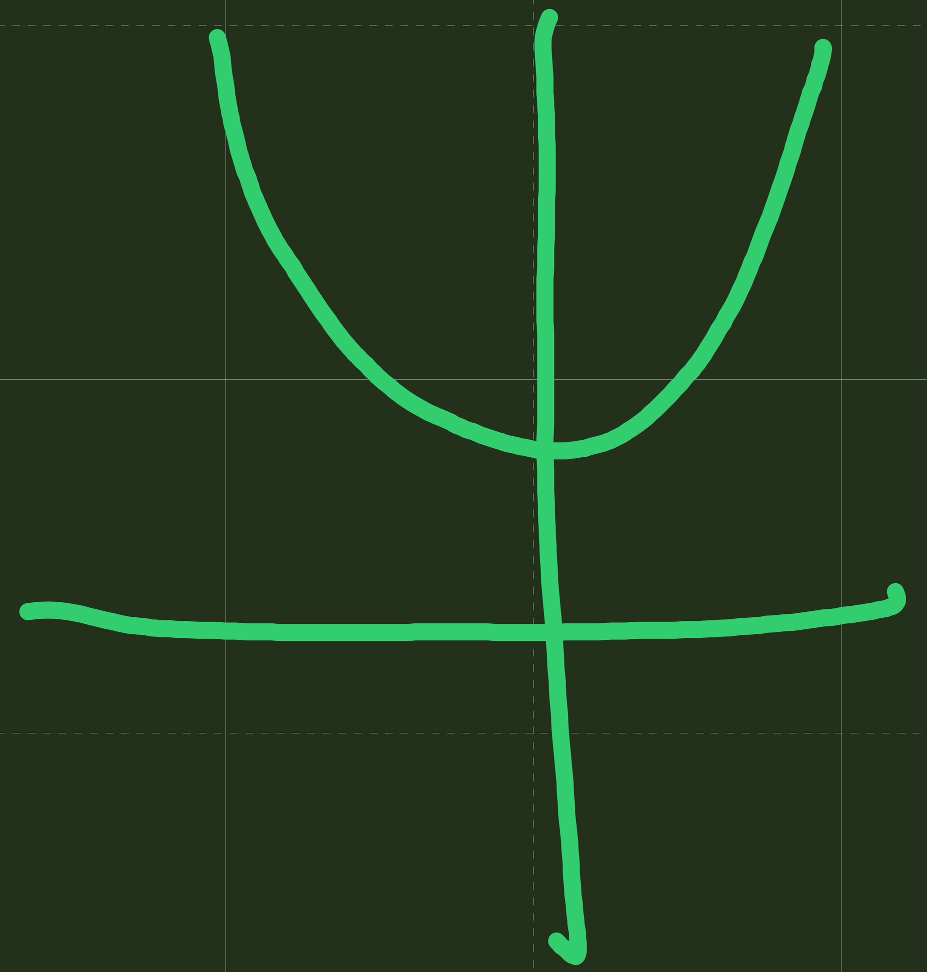
$$\left(\frac{p}{2}\right)^2-q$$

Diskriminante D

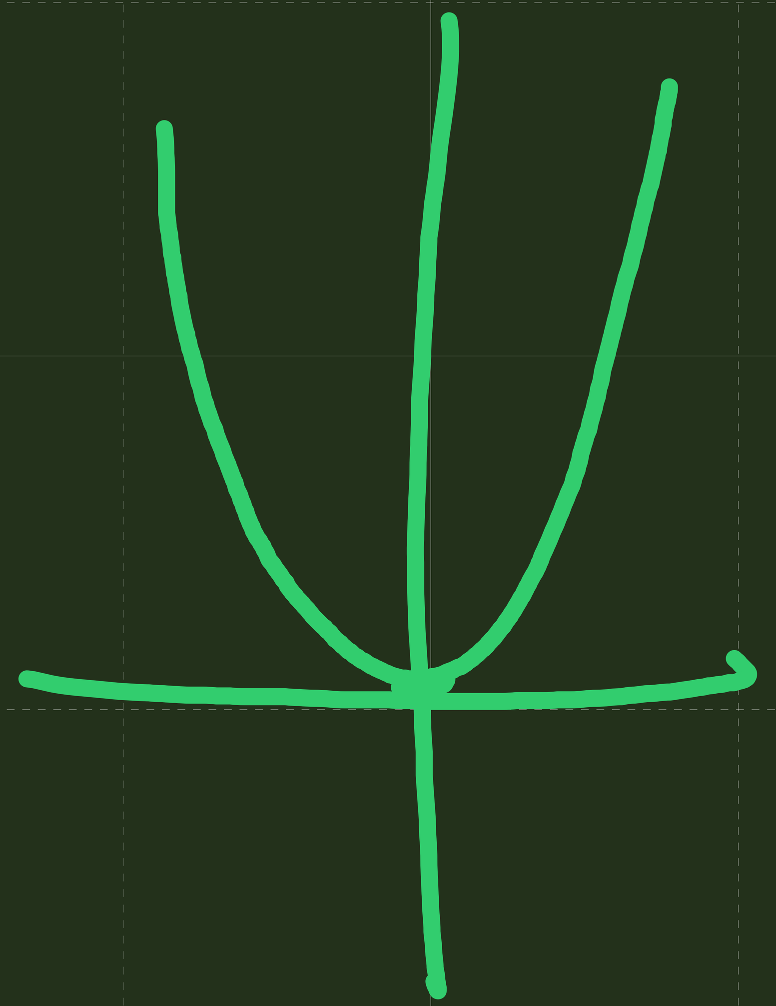
$D<0$ $\mathbb{L}=\{\}$

$D=0$ $x=-\frac{p}{2}$

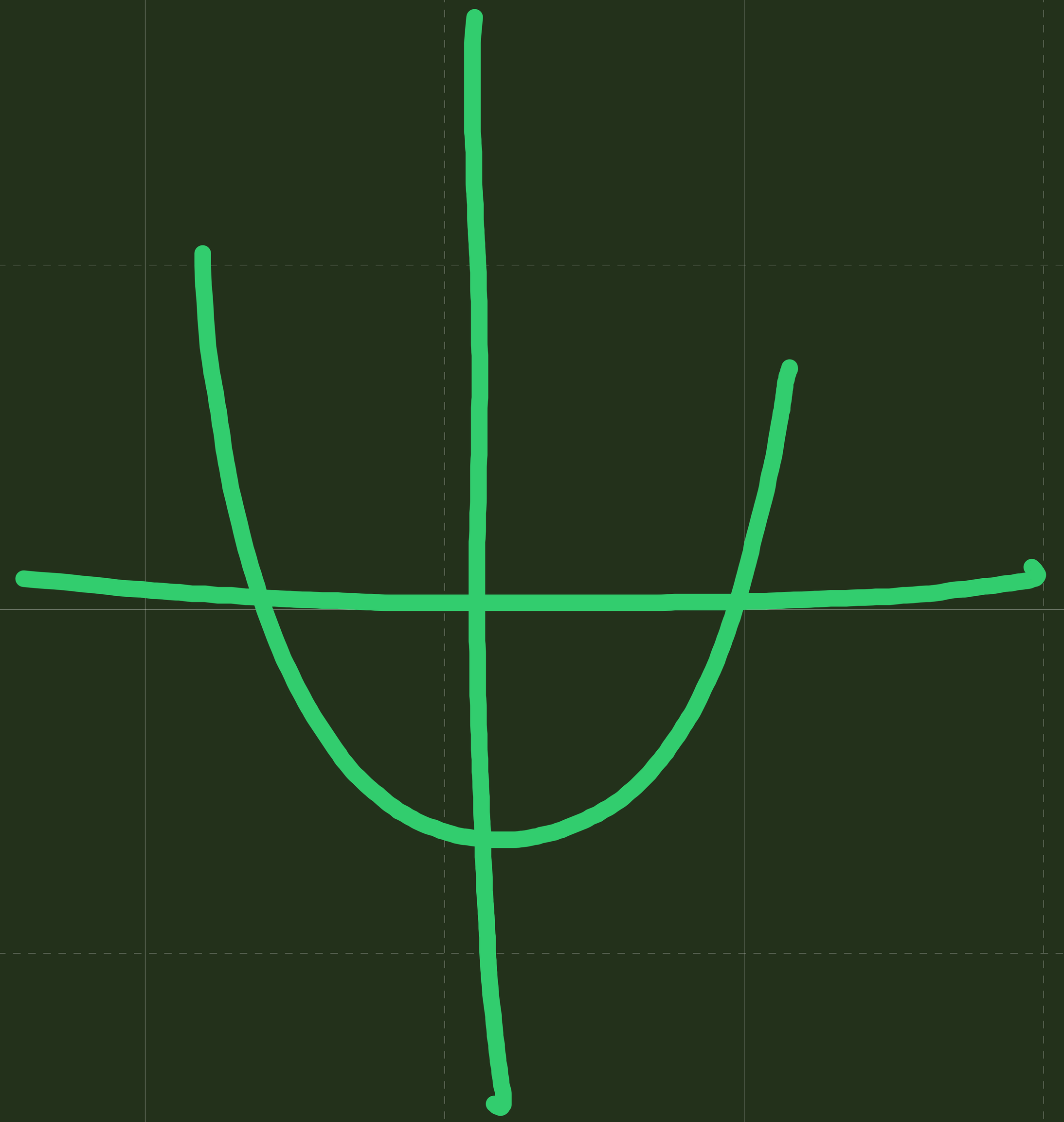
$D>0$ 2 Lösungen



keine Nullstelle



genau eine Nullstelle



2 Lösungen

Beispiel $0 = 2x^2 + 8x + 6 \quad | :2$

$0 = x^2 + 4x + 3$ Normalform

$p = 4 \quad q = 3$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 3}$$

$$= -2 \pm \sqrt{4 - 3}$$

$$= -2 \pm \sqrt{1}$$

$$= -2 \pm 1$$

$x_1 = -3$

$x_2 = -1$

Probe:

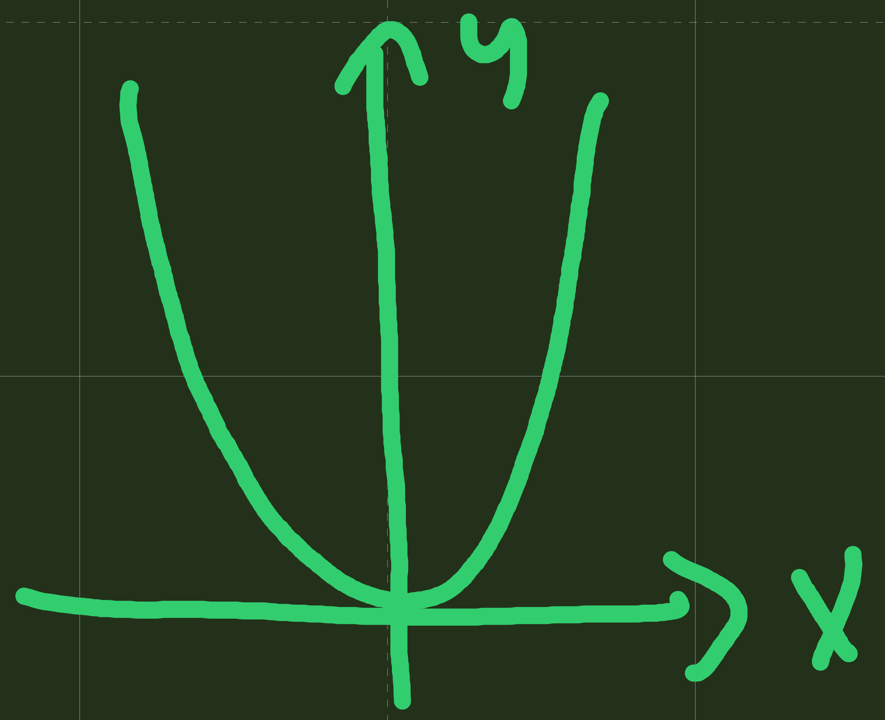
$2 \cdot (-3)^2 + 8 \cdot (-3) + 6$

$2 \cdot 9 \quad - 24 + 6$

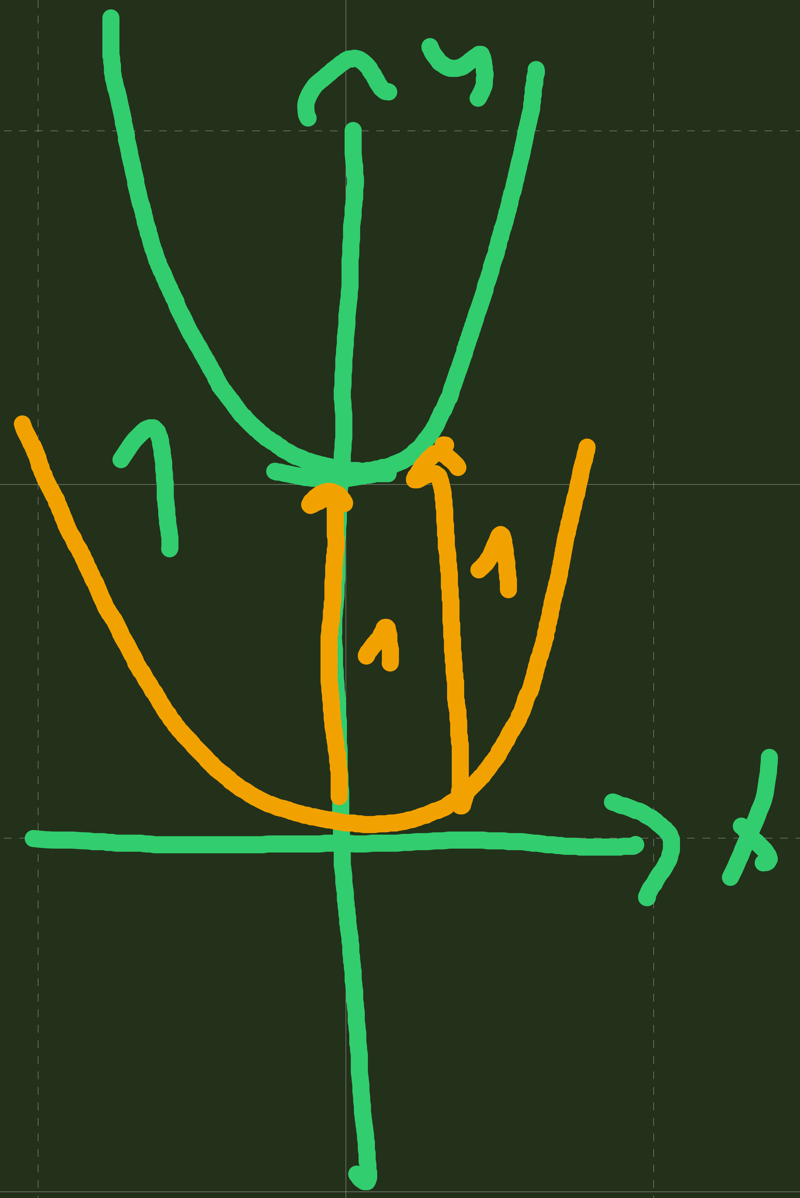
$18 - 24 + 6 = 0 \checkmark$

Hoch-/Tiefpunkte = Scheitelpunkte

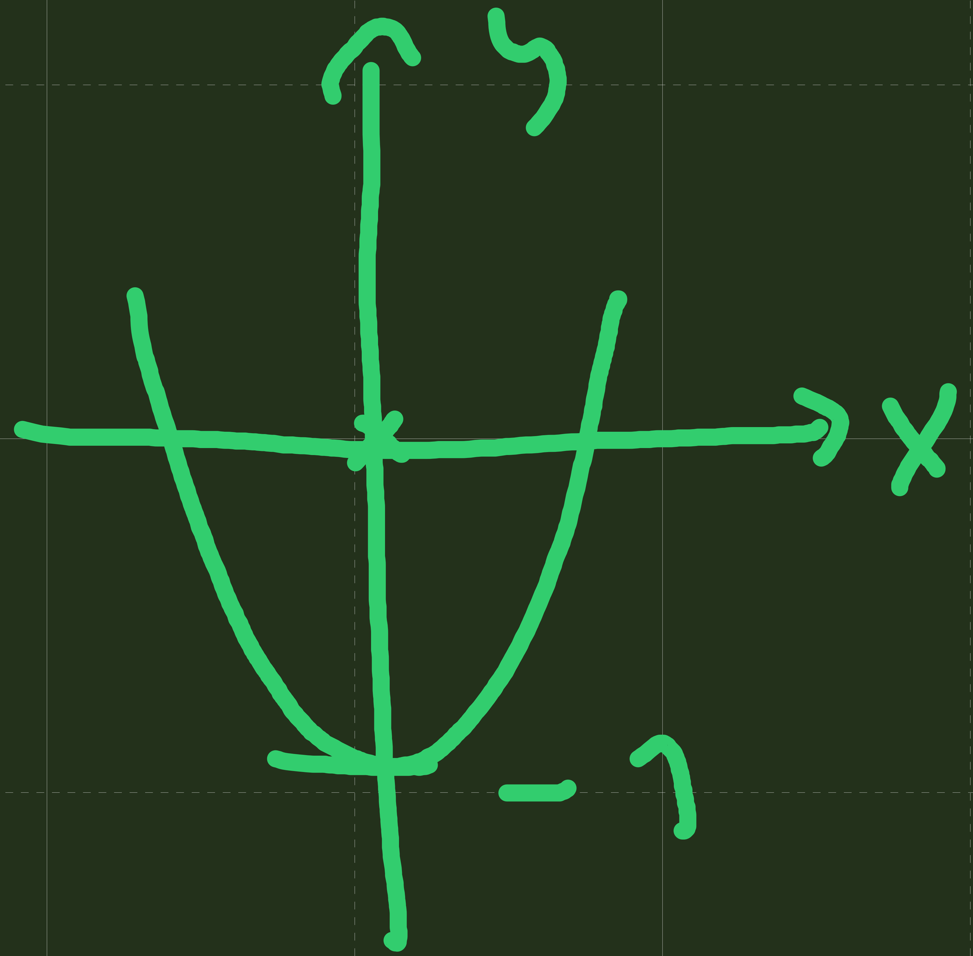
Maximum / Minimum



$y = x^2$



$y = x^2 + 1$



$y = x^2 - 1$

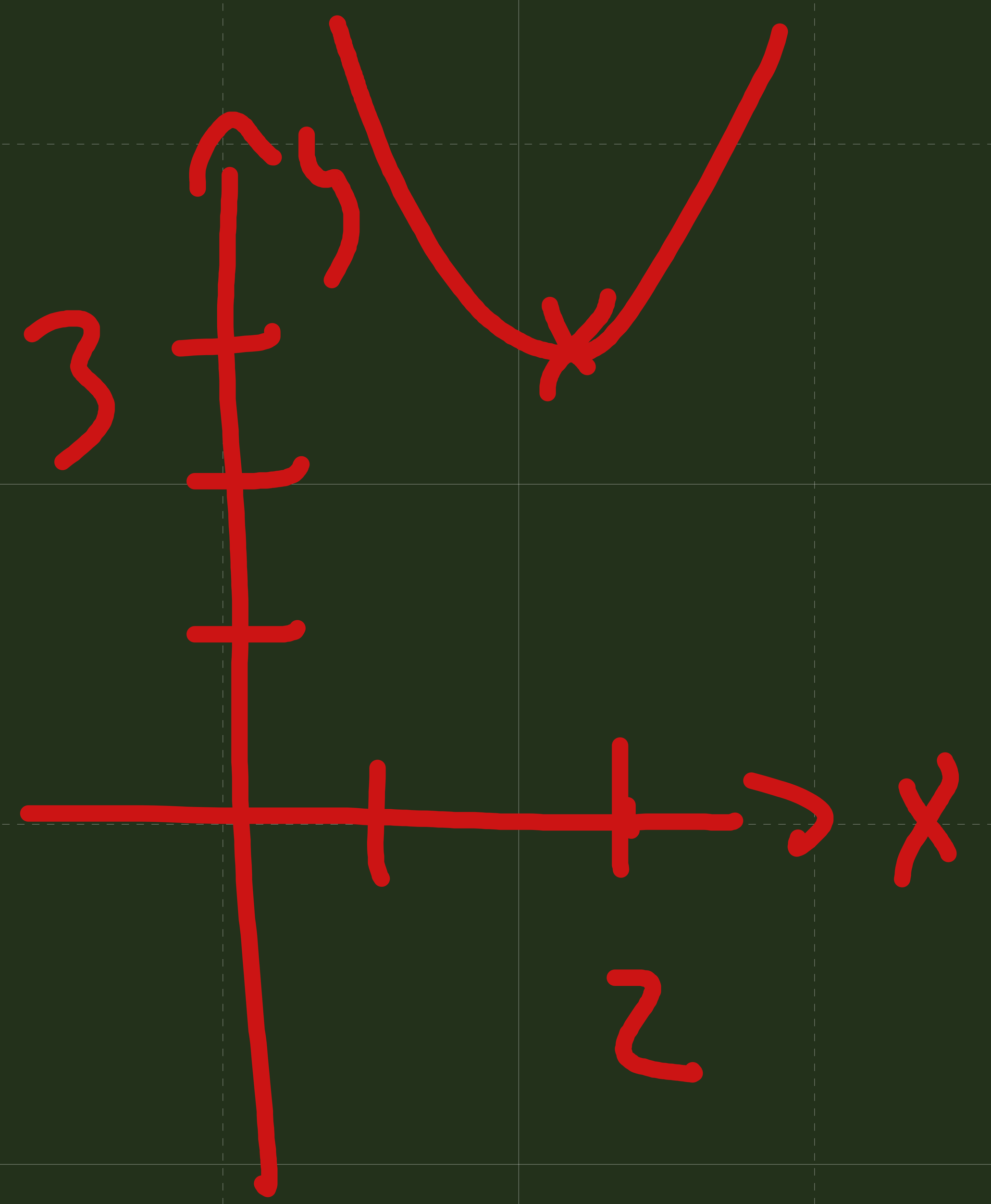
$y = (x - (+1))^2$ $y = (x - (-1))^2$

$y = (x - x_s)^2$

Scheitelpunktform $y = (x - x_S)^2 + y_S$

Beispiel: $y = (x - (+2))^2 + 3$

$S(2|3)$



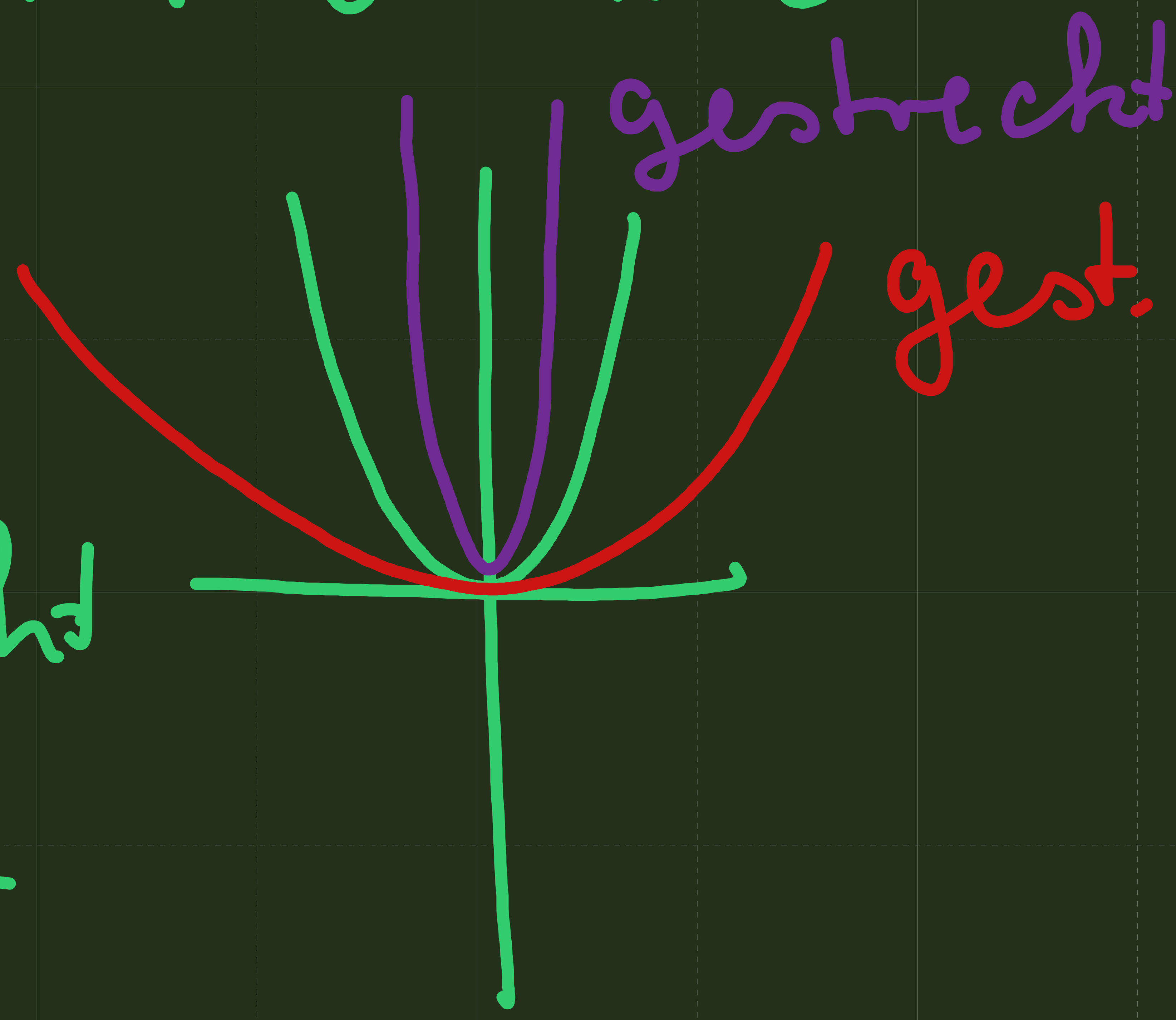
Stauchen u. Strecken der Normalparabel:

$a x^2$

$\uparrow$

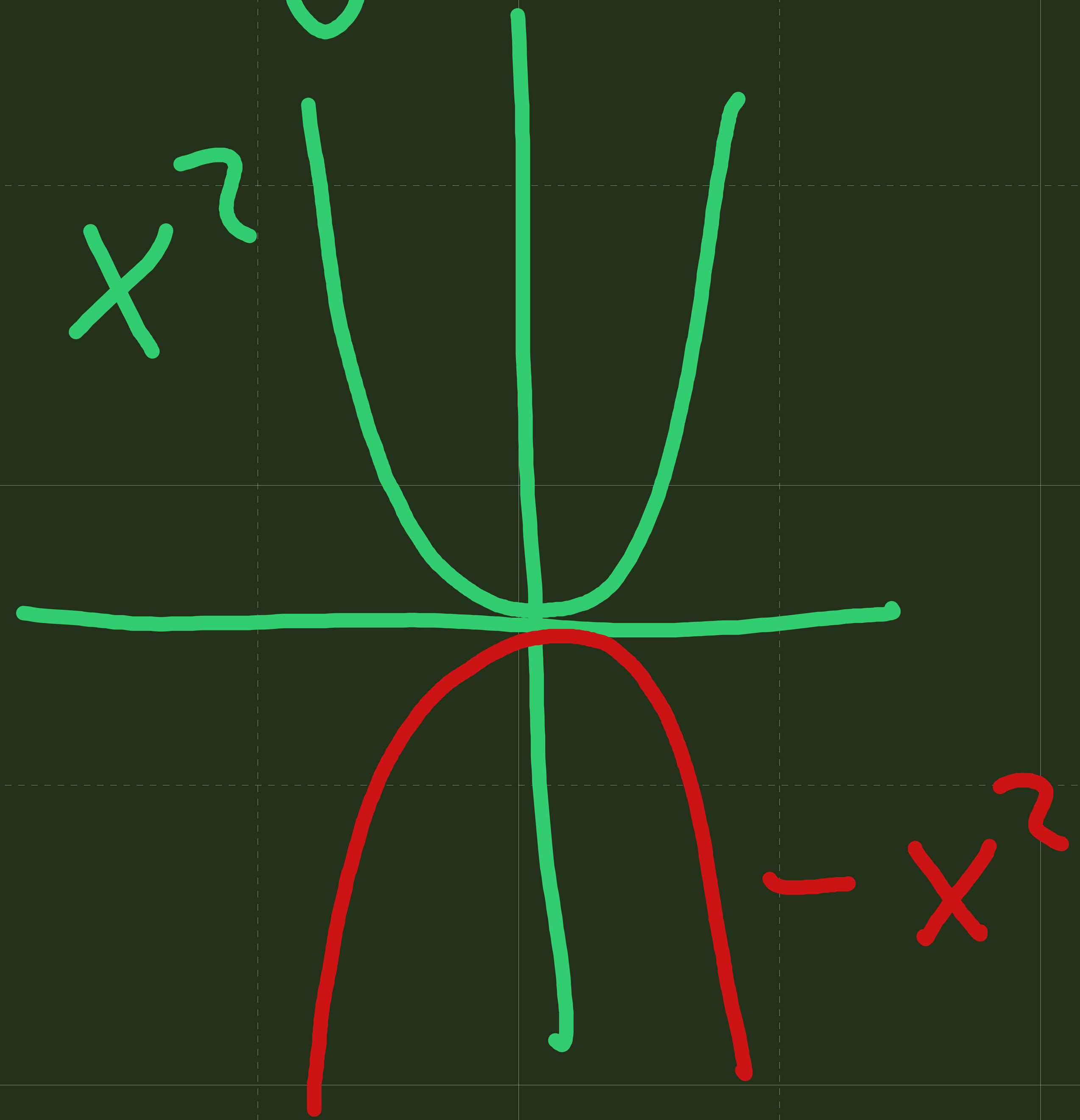
$0 < a < 1$ gestaucht

$a > 1$ gestreckt



Spiegelung an der x-Achse

$y = -x^2$



$$y = 2x^2 + 8x + 6$$

Ziel: $y = a \cdot (x - x_S)^2 + y_S$

$$y = 2 \cdot (x^2 + 4x + 3)$$

(1) Faktor vor x^2 ausklammern

$$y = 2 \cdot ((x^2 + \underbrace{4}_{:2}x + 2^2) \underbrace{- 2^2}_{} + 3)$$

$$\underbrace{+2^2 - 2^2}_{=0}$$

(2) Faktor vor x durch 2 dividieren. Das Quadrat davon addieren u. subtrahieren

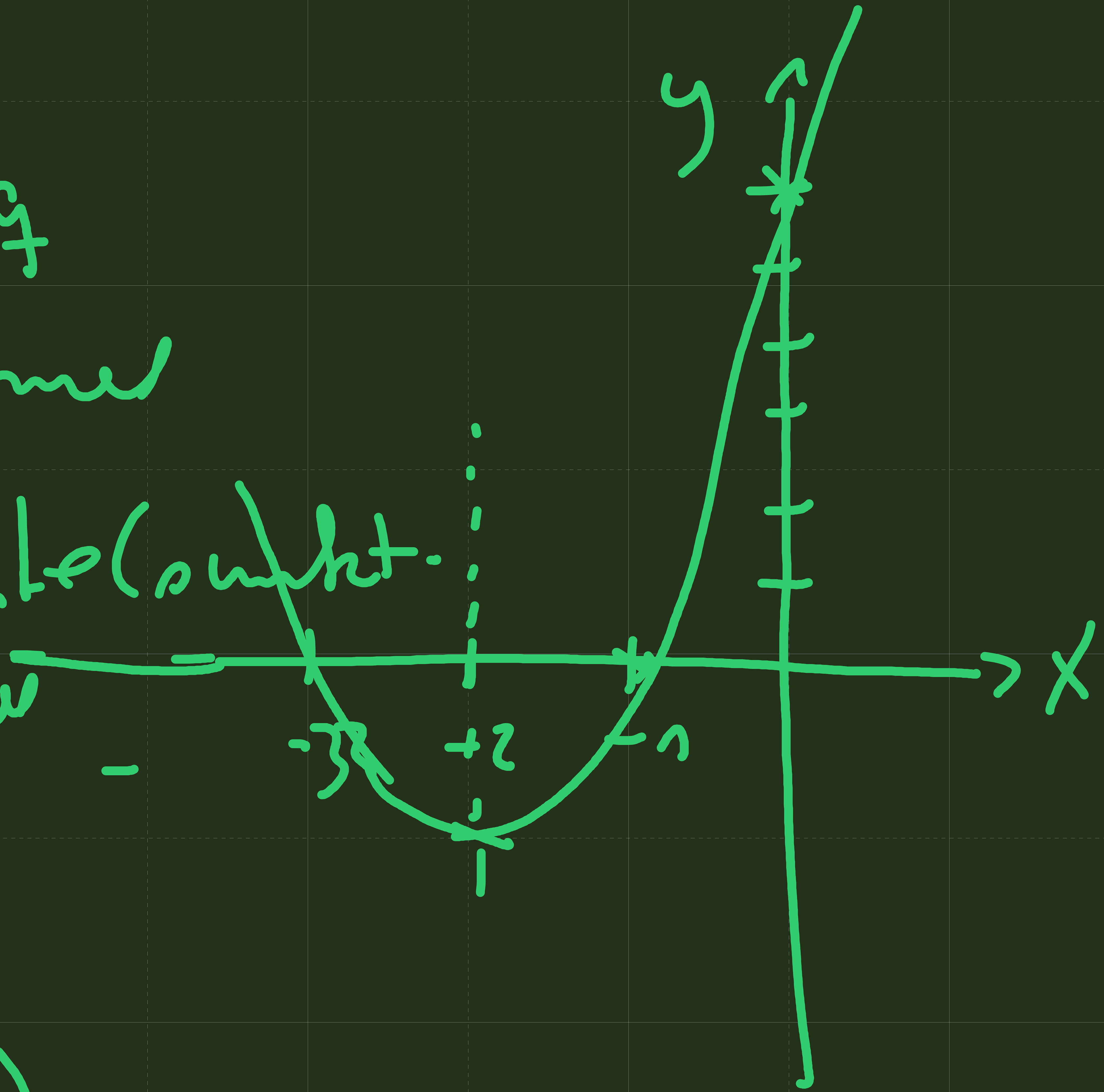
$$y = 2((x+2)^2 - 1)$$

$$y = 2(x+2)^2 - 2$$

$$2(x-(-2))^2 - 2 \qquad S(-2|-2)$$

$N_1(-1|0)$
$N_2(-3|0)$ } p-q Formel

$S(-2|-2)$ } Scheitelpunkt-formel



$y = 2x^2 + 8x + 6$

S(0|6)

$2x^2+8x+6=0$

$x^2+4x+3=0$

$-2 \pm \sqrt{4-3}$

-2 ± 1

$-3 \quad -1$

1) $3x^2 + 24x + 21 = 0 \quad | :3$

$x^2 + 8x + 7 = 0 \qquad p = 8 \quad q = 7$

$$-\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x_{1,2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - (+7)}$$

$$-4 \pm \sqrt{16 - 7}$$

$$-4 \pm \sqrt{9}$$

$$-4 \pm 3$$

$x_1 = -7$

$x_2 = -1$

Probe: $x = -1$

$3 - 24 + 21 = 0$ ✓

$$\frac{1}{3}x^2 + \frac{1}{6}x - \frac{1}{6} = 0 \quad \Big| : \frac{1}{3} \quad \text{d.h.} \cdot 3$$

$$x^2 + \frac{1}{2}x - \frac{1}{2} = 0 \qquad p = \frac{1}{2} \quad q = -\frac{1}{2}$$

$$x_{1,2} = -\frac{\left(\frac{1}{2}\right)}{2} \pm \sqrt{\left(\frac{\left(\frac{1}{2}\right)}{2}\right)^2 - \left(-\frac{1}{2}\right)} \qquad \frac{\frac{1}{2}}{2} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{2}{1}\right)} = \frac{1}{1} \cdot \frac{1}{2} = \frac{1}{4}$$

$$= -\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{8}{16}} \qquad x_{1,2} = -\frac{1}{4} \pm \frac{3}{4}$$

$$= -\frac{1}{4} \pm \sqrt{\frac{9}{16}} \qquad x_1 = -1 \quad x_2 = \frac{1}{2}$$

$$2x^2 + 14x + 20 = 0 \quad | :2$$

$$\rightarrow x^2 + 7x + 10 = 0 \leftarrow$$

$$x_{1,2} = -\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 - 10}$$

$$-\frac{7}{2} \pm \sqrt{\frac{49}{4} - \frac{40}{4}}$$

$$-\frac{7}{2} \pm \sqrt{\frac{9}{4}}$$

$$-\frac{7}{2} \pm \frac{3}{2}$$

$x_1 = -5$

$x_2 = -2$

$\rightarrow x_1 \cdot x_2 = 10$

$\rightarrow x_1 + x_2 = -7$

$x_1 \cdot x_2 = q$

$x_1 + x_2 = -p$

Satz von Vieta für Fanle (= Ch)

Definitionsbereich: Welche Werte darf ich für x einsetzen

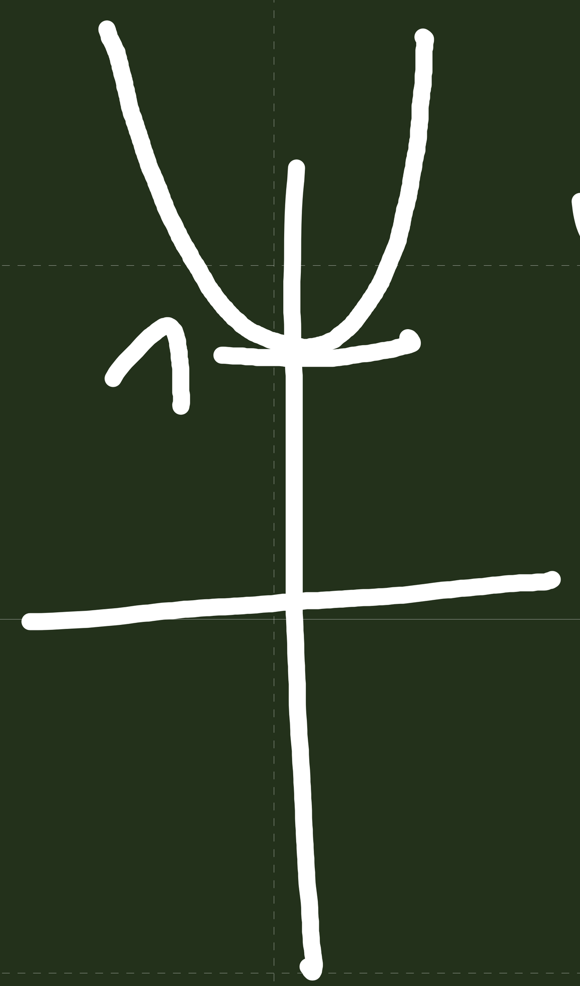
$y = x^2 + 7x + 3 \qquad \mathbb{D} = \mathbb{R}$ (Reellen Zahlen)

3 Ausnahmen

1. $y = \frac{1}{x} \qquad \mathbb{D} = \mathbb{D} \setminus \{0\}$ x im Nenner
2. $y = \sqrt{x} \qquad \mathbb{D} : \mathbb{R}_0^+$ x unter der Wurzel
3. $y = \log$ ↯

Werteberich: Welche Werte nimmt y an

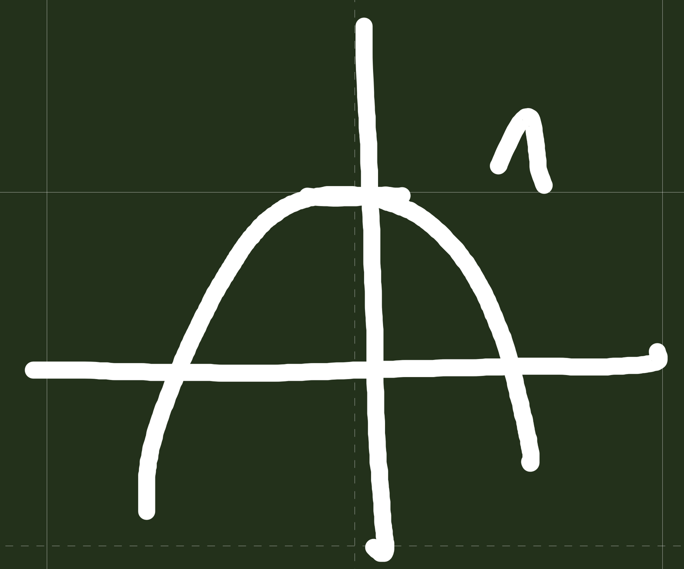
① durch HP/TP



$y = x^2+1$ TP$(0|1)$

$W = [1; \infty)$

$[1; \infty[$

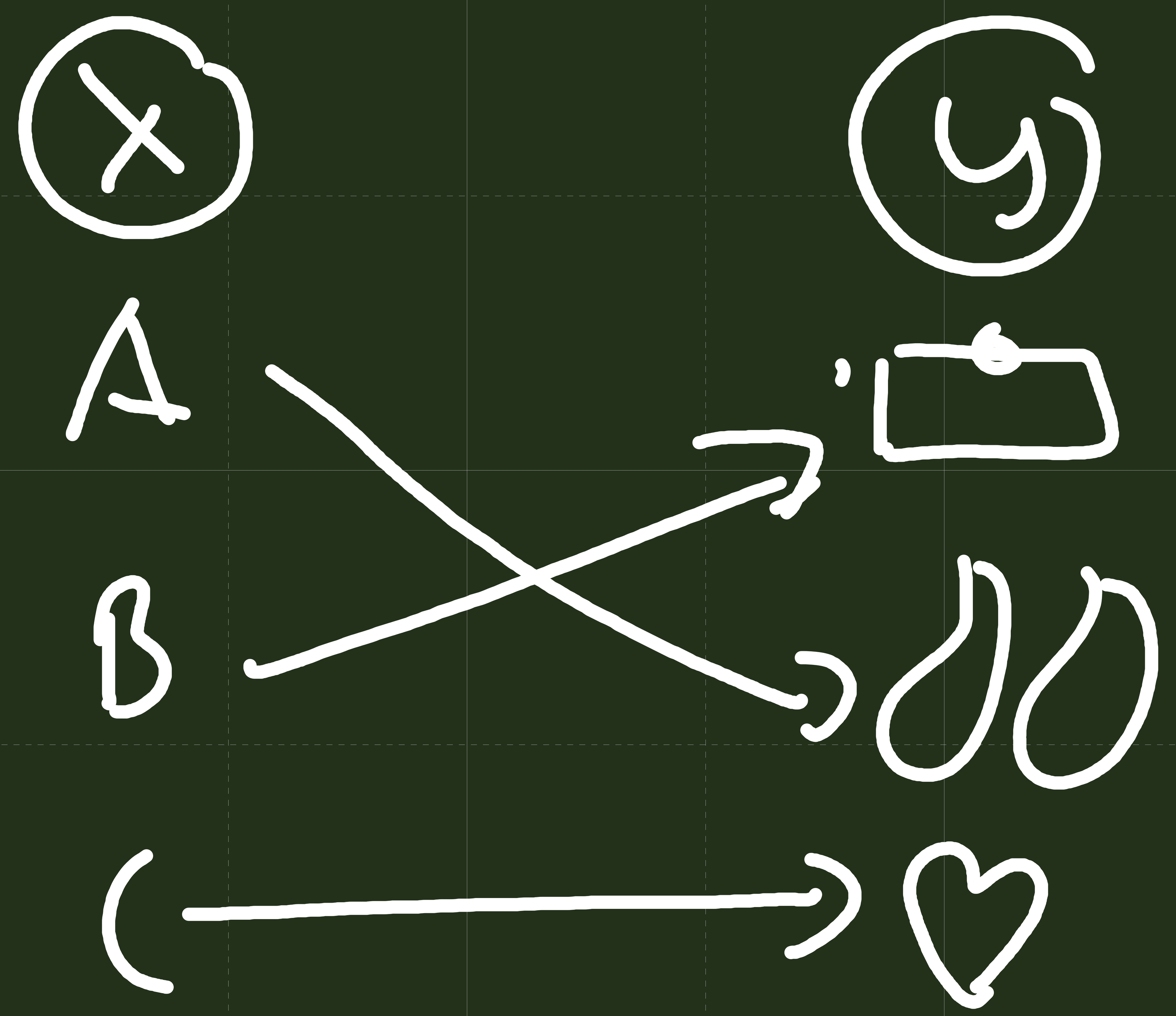


$y = -x^2+1$ HP$(0|1)$

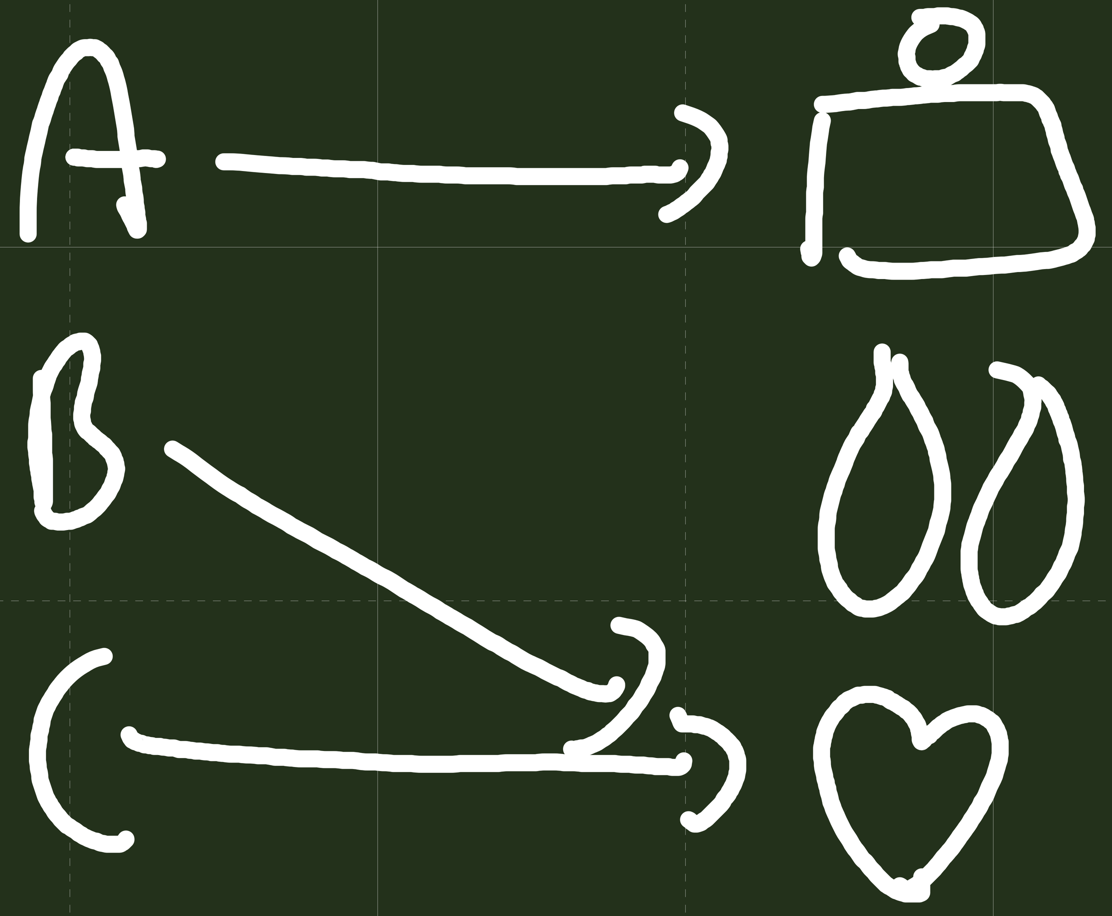
$W =]-\infty; 1]$

2. Möglichkeit: Über den Definitionsbereich
der inversen Funktion
Erklärung später

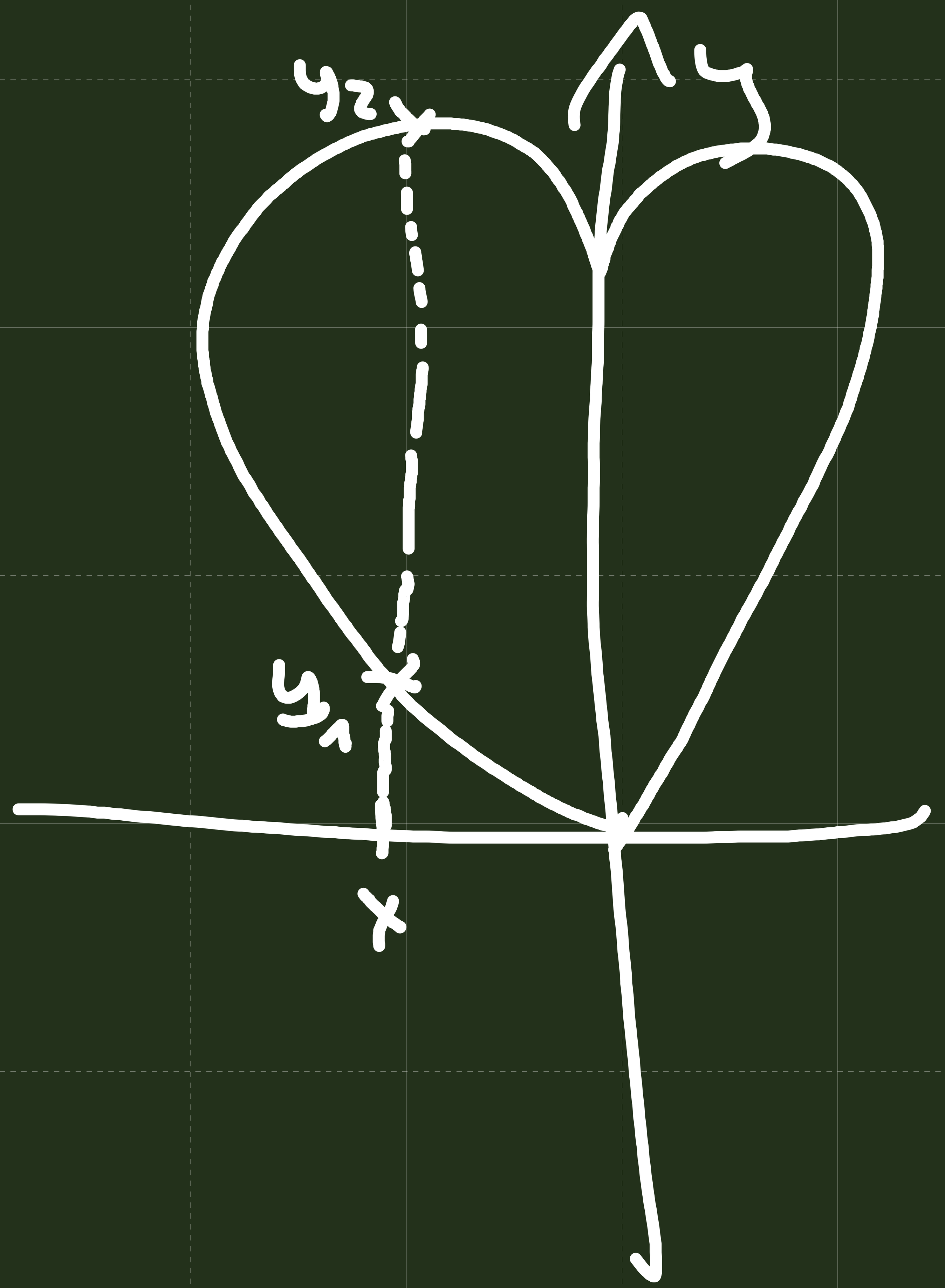
Bijektiv:

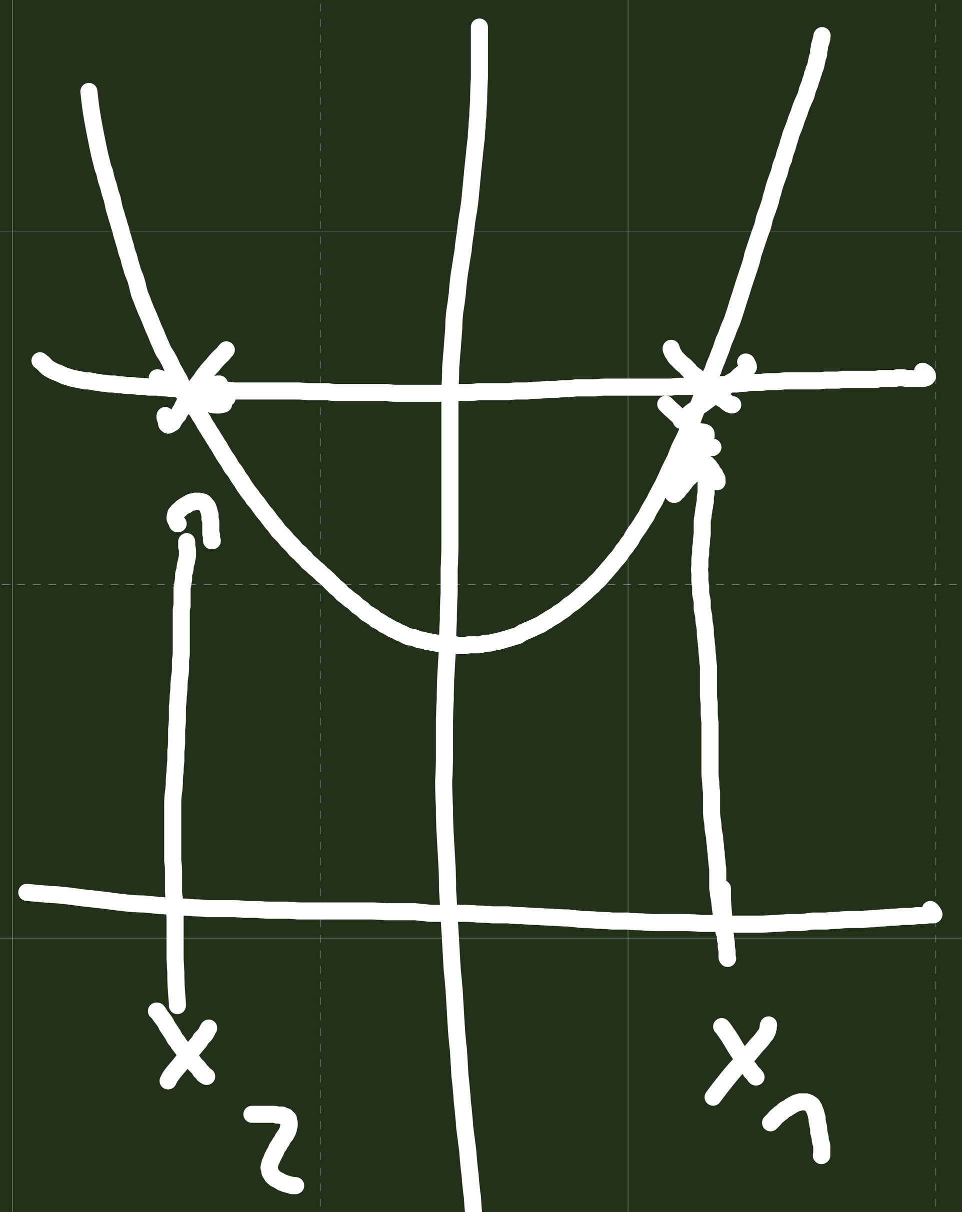


bijektiv



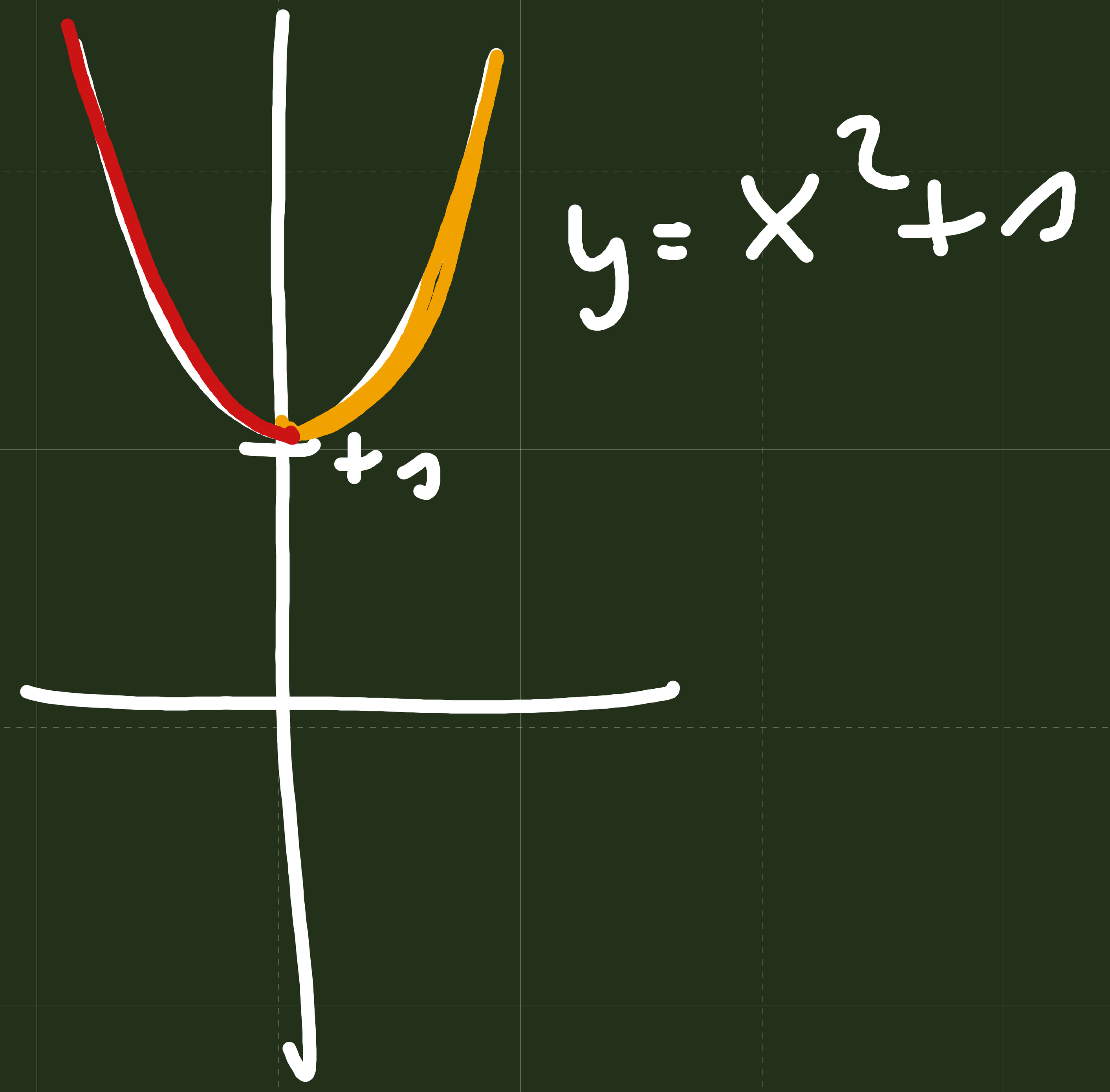
nicht bijektiv





bijektiv?

bijektive Funktionseinschränkung?



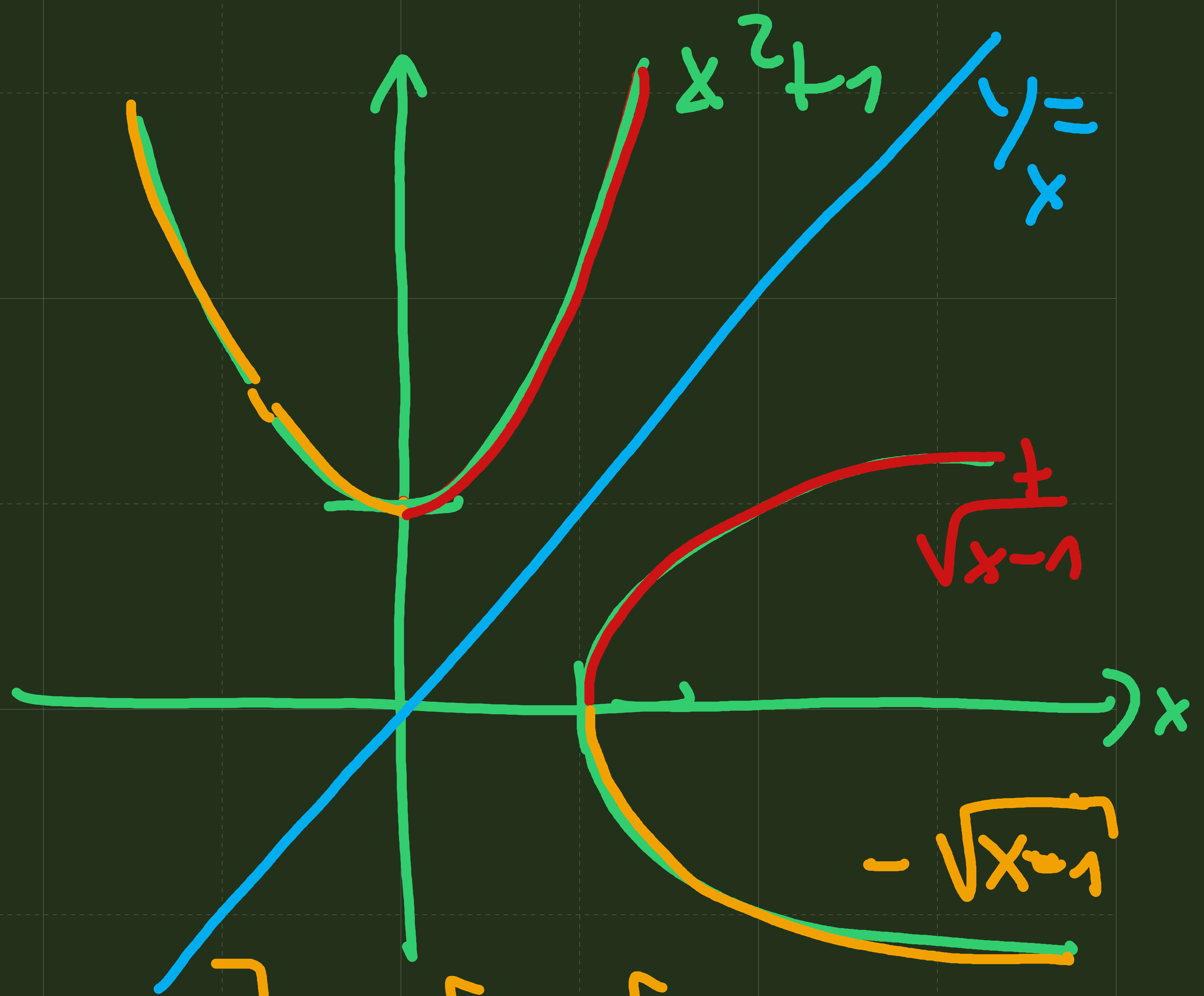
Inverse

$y = x^2+1 \quad |\ \text{V.T}$

$x = y^2+1 \quad |\ -1$

$x-1 = y^2 \quad |\ \sqrt{\ }$

$\pm\sqrt{x-1} = y_{1,2}$



oberT $\quad D_1 =]-\infty;0] \quad W_1 = [1;+\infty[$

$D_2 = [0;+\infty[\quad W_2 = [1;+\infty[$

Intervall

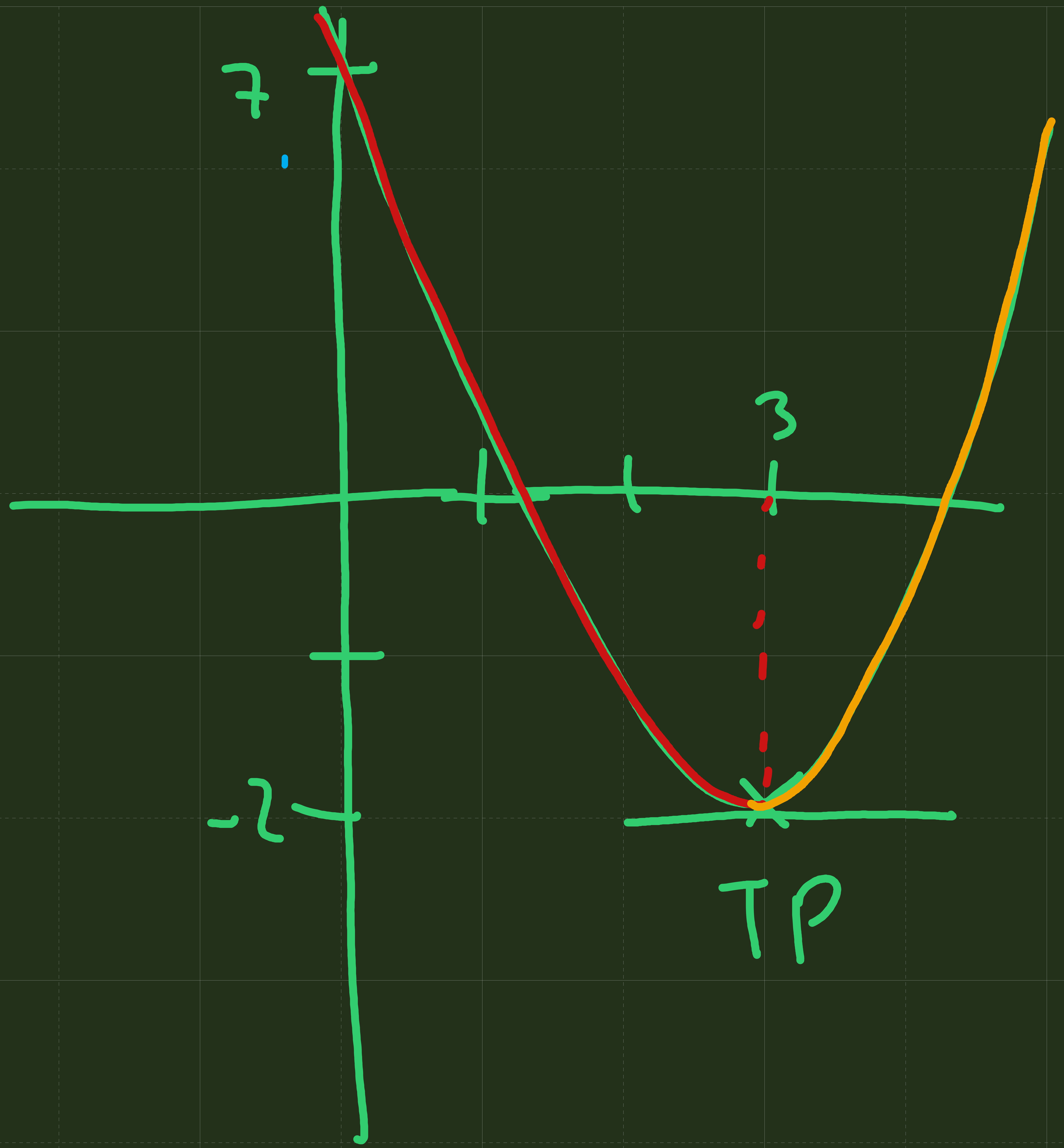
$[2;4]$	$2 \leq x \leq 4$	abgeschloss.
$]2;4]$	$2 < x \leq 4$	halboffen
$[2;4[$	$2 \leq x < 4$	halboffen
$]2;4[$	$2 < x < 4$	offene I.

Intervall

0. Scheitelpunkt bestimmen.

1. Zeichnen Sie den Graphen $y = x^2 - 6x + 7$

$$y = 1x^2 - 6x + 7$$
$$= x^2 - 6x + 3^2 - 3^2 + 7$$
$$= (x-3)^2 - 2 \qquad TP(3|-2)$$



$\mathbb{D} = \mathbb{R}$

$\mathbb{W} = [-2; +\infty[$

$\mathbb{D}_1 =]-\infty; 3] \quad \mathbb{W}_1 = [-2; +\infty[$

$\mathbb{D}_2 = [3; +\infty[\quad \mathbb{W}_2 = [-2; +\infty[$

$$y = x^2 - 6x + 7 \quad | \text{ V.T.}$$

$$x = y^2 - 6y + 7 \quad | -x$$

$$0 = y^2 \boxed{-6} y + 7 - x \quad | \; p = -6$$

$$q = 7 - x$$

$$y_{1,2} = 3 \pm \sqrt{9 - (7 - x)}$$

$$3 \pm \sqrt{9 - 7 + x}$$

$$y_{1,2} = 3 \pm \sqrt{2 + x} \qquad \mathbb{D} = [-2; +\infty[$$