Potenzen von Summen

$$(a+b)^0 = 1 = 1 \cdot a^0 \cdot b^0$$

$$(a+b)^1 = a+b = 1 \cdot a^1 \cdot b^0 + 1 \cdot a^0 \cdot b^1$$

$$(a+b)^2 = a^2 + 2ab + b^2 = 1 \cdot a^2 \cdot b^0 + 2 a^1 \cdot b^1 + 1 \cdot a^0 \cdot b^2$$

$$(a+b)^3 = 1a^3b^0 + 3a^2b^1 + 3a^1b^2 + 1a^0b^3$$

$$(a+b)^4 = 1a^{\boxed{4}}b^{\boxed{0}} + 4a^{\boxed{3}}b^{\boxed{1}} + 6a^{\boxed{2}}b^{\boxed{2}} + 4a^{\boxed{1}}b^{\boxed{3}} + 1a^{\boxed{0}}b^{\boxed{4}}$$

$$(a+b)^n = ?$$

$$(a+b)^3 = (a+b)^2 \cdot (a+b) =$$

$$(a^2+2ab+b^2)\cdot(a+b) =$$

$$a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3 =$$

$$a^3 + 3a^2b + 3ab^2 + b^3 =$$

$$1\cdot a^3 \cdot b^0 + 3a^2b^1 + 3a^1b^2 + 1a^0\cdot b^3$$

$$(a+b)^4 = (a+b)^3 \cdot (a+b) =$$

$$(a^3+3a^2b+3ab^2+b^3)\cdot(a+b) =$$

$$a^4+a^3b+3a^3b+3a^2b^2+3a^2b^2+3ab^3+ab^3+b^4 =$$

$$a^4+4a^3b+6a^2b^2+4ab^3+b^4 =$$

$$1a^4b^0+4a^3b^1+6a^2b^2+4a^1b^3+1\cdot a^0b^4$$

$$(a+b)^4 = 1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

$$\begin{array}{ccccc} & 1 & 3 & 3 & 1 \\ 1 & 3 & 3 & 1 & \\ \hline 1 & 4 & 6 & 4 & 1 \end{array}$$

$$\binom{4}{0} = 1$$

$$\binom{4}{1} = \frac{4}{1!} = \frac{4}{1} = 4$$

$$\binom{4}{2} = \frac{4\cdot 3}{2!} = \frac{4\cdot 3}{1\cdot 2} = 6$$

$$\binom{4}{3} = \frac{4\cdot 3\cdot 2}{3!} = \frac{4\cdot 3\cdot 2}{1\cdot 2\cdot 3} = 4$$

$$\binom{4}{4} = \frac{4\cdot 3\cdot 2\cdot 1}{4!} = \frac{4\cdot 3\cdot 2\cdot 1}{1\cdot 2\cdot 3\cdot 4} = 1$$

Pascalsche Dreieck

1

1 1

$(a+b)^2$ 1 2 1

+

$(\ )^3$ 1 3 3 1

$(\ )^4$ 1 4 6 4 1

Koeffizienten für $(a+b)^4$

$(\ )^5$ 1 5 10 10 5 1

Koeffizienten für $(a+b)^5$

1 6 15 20 15 6 1

$$(a+b)^{\boxed{4}} = 1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0\cdot b^4$$

$$\binom{4}{0}\begin{matrix}=n\\=k\end{matrix} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

$$\boxed{(a+b)^{\boxed{n}} = \sum_{k=0}^{n} \binom{n}{\boxed{k}} \cdot a^{n-k} \cdot b^{\boxed{k}}}$$

Binomischer Lehrsatz

n ist eine natürliche Zahl

$k=0 \mid k=1 \mid k=2 \mid k=3 \mid k=4$

$n-k=4 \quad n-k=3 \quad n-k=2 \quad n-k=1 \quad n-k=0$

Exkurs Summenzeichen

$$0 + 1 + 2 + 3 + 4 = \sum_{k=0}^{4} k$$

$$(0, 1, 2, 3, 4)$$

$$\begin{array}{ccccccccccc}
0 & + & 2 & + & 4 & + & 6 & + & 8 & = & \sum\limits_{k=0}^{4} 2 \cdot k \\
\uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
k=0 & & k=1 & & k=2 & & k=3 & & k=4 & & \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
0 & + & 1 & + & 4 & + & 9 & + & 16 & = & \sum\limits_{k=0}^{4} k^2
\end{array}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot a^{n-k} \cdot b^k$$

$$(2x+4y)^3$$

$a = 2x \quad b = 4y \quad n = 3$

$$= \sum_{k=0}^{3} \binom{3}{k} \cdot (2x)^{3-k} \cdot (4y)^k =$$

$$\binom{3}{0} \cdot (2x)^3 \cdot (4y)^0 + \binom{3}{1} \cdot (2x)^2 \cdot (4y)^1 + \binom{3}{2} \cdot (2x)^1 (4y)^2 + \binom{3}{3} (2x)^0 \cdot (4y)^3$$

$$= 1 \cdot 8 \cdot x^3 \cdot 1 + 3 \cdot 4x^2 \cdot 4y + 3 \cdot 2x \cdot 16y^2 + 1 \cdot 1 \cdot 64y^3$$

$$= 8 \cdot x^3 + 48 \cdot x^2 \cdot y + 96\,xy^2 + 64 \cdot y^3$$

$$\binom{3}{0} = 1$$

$$\binom{3}{1} = \frac{3}{1} = 3$$

$$\binom{3}{2} = \frac{3 \cdot 2}{1 \cdot 2} = 3$$

$$\binom{3}{3} = \frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3} = 1$$

$$(2x+4y)^3 = (2x+4y)\cdot(2x+4y)\cdot(2x+4y) =$$

$$(4x^2+8xy+8xy+16y^2)\cdot(2x+4y) =$$

$$(4x^2+16xy+16y^2)\cdot(2x+4y) =$$

$$8x^3+16x^2y+32x^2y+64xy^2+32xy^2+64y^3 =$$

$$8x^3+48x^2y+96xy^2+64y^3 \quad \checkmark$$

Kontrolle per Ausmultiplizieren