Small Worlds and Their Modelling

A TERM PAPER BY CARINA KIRSCHNER

WS 2018/19

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Introduction

https://www.shutterstock.com/search/society

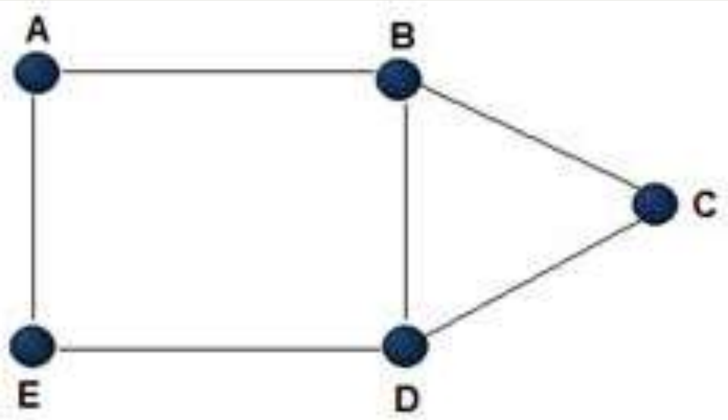
https://steemit.com/steemit/@o1o1o1o/cap-theorem-for-social-networks

Six Degrees of Separation

Degree of Separation

The least amount of links between two nodes

Example:



E and C have two Degrees of Separation

Stanley Milgram's Chain Letter Experiment

"How many acquaintances do you need to connect any two persons in the United States with each other?"

Stanley Milgram

Stanley Milgram's Chain Letter Experiment

Stanley Milgram's Chain Letter Experiment

How to take part in this Study

1. Add your name to the roster at the bottom of this sheet.
2. Detach one postcard. Fill it out and return it to Harvard University.
3. If you know the target person on a personal basis, mail this folder directly to her (him).
4. If you don't know the target person on a personal basis, do not try to contact him directly. Instead, mail this folder (postcards and all) to a personal acquaintance who is more likely than you to know the target person.

Stanley Milgram's Chain Letter Experiment

Median of intermediate persons

5.5

Other Degrees of Separation

Degree of Separation between pages in the WWW is **19**

Degree of Separation on molecules in the cell is **3**

Degree of Separation between routers in the internet is **10**

Why is the Degree of Separation very low?

- Each node has on average k links
- With one Step we can reach k other nodes
- k^2 nodes are two links away -> k^d nodes are d links away

Formula to calculate the average Separation with $k^d = N$

$$d = \frac{\log N}{\log k}$$

Ways of Modelling

- Random Networks
- Small World Networks
- Scale Free Networks

Random Networks

Random Networks

G(n, p)

1. Nodeset(v1,…,vn) owns n nodes.
2. The Node vi and vj with i < j are linked with the probability p. This happens separately for different pairs of nodes.

Random Networks

If p is set and n grows towards infinity:

- The nodes vi and vj are linked with the probability p.
- The probability that there is a way of the length 2 between two nodes depends on n.
- There are n-2 ways of the length 2.
- The probability that the nodes are in the random network is p^2.
- The probability that **none** of the n-2 ways are in the random network is

$$(1-p^2)^{n-2}$$

- For $n \to \infty$ the probability goes towards 0.
- For $n \to \infty$ the probability that a pair of nodes is linked through a way of 2 goes towards 1.

Random Networks

$p = \frac{c}{n}$ where $c > 0$

The average amount of links at one node would be:

$(n-1)\,p = \frac{(n-1)\,c}{n}$

For n → ∞ the term $\frac{(n-1)}{n}$ goes towards 1.

For big n the amount of links will be c.

Random Networks

G$(n, \frac{c}{n})$ with $c = 1000$ and $n = 7\,000\,000\,000$

Every pair of nodes knows each other with the probability $\frac{c}{n} = \frac{1}{7\,000\,000}$

Under 1000 acquaintances are $\frac{1000 \cdot 999}{2} \approx 500\,000$ possibilities to choose 2 persons

Each of those 500 000 pairs has the probability $\frac{1}{7\,000\,000}$ to know each other.

The expected amount of acquaintances who know each other is $\frac{500\,000}{7\,000\,000} \approx 0.071$

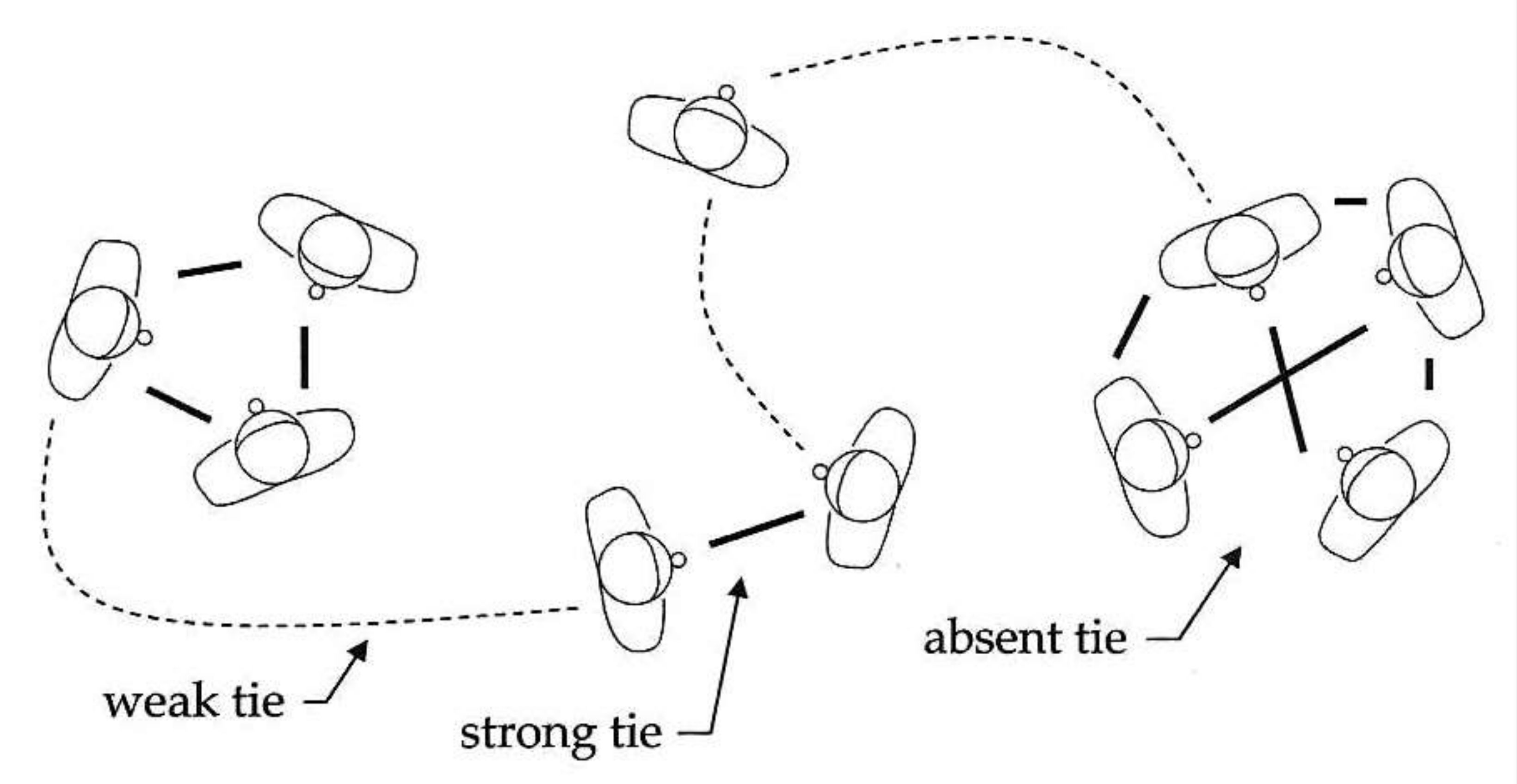
Small World Networks

Small World Networks

“How big is the chance that my two best friends know each other? ”

Duncan Watts

Small World Networks



https://www.leadershipcloseup.com/2012/12/14/strength-of-weak-ties-in-social-networking-seek-to-be-worth-knowing/

Small World Networks

The **clustering coefficient** C for a Node is given by the amount of links between the nodes within its neighborhood divided by the number of links that could possibly exist between them.

Small World Networks – The Erdös Number

- **Paul Erdös** (*26th March 1913, Budapest - †20th September, Warschau)
- Very famous and eccentric mathematician of the 20th Century
- He published over **1500** papers with **507** coauthors.

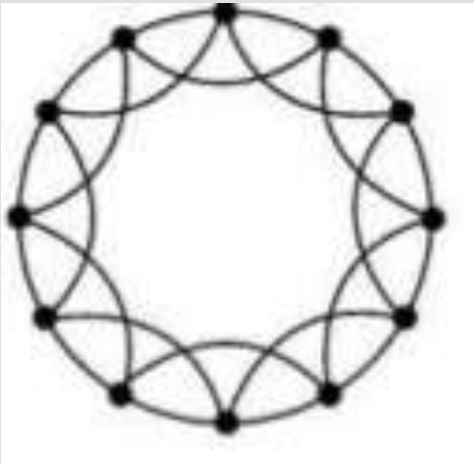
https://www.leadershipcloseup.com/2012/12/14/strength-of-weak-ties-in-social-networking-seek-to-be-worth-knowing/

Small World Networks - The Erdös Number

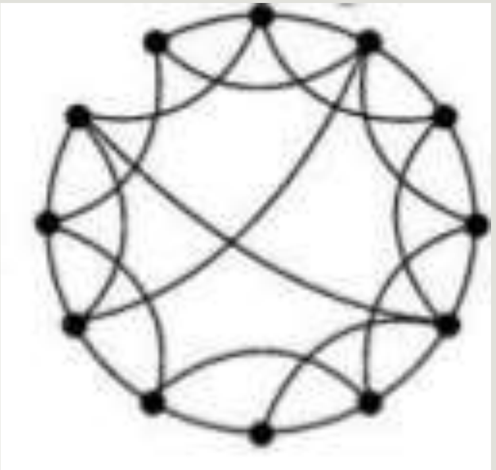
Erdös Number	Number of Mathematicians
0	1
1	502
2	5 713
3	26 422
4	62 163
5	66 157
6	32 280
7	10 431
8	3 214
9	953
10	262
11	94
12	23
13	4
14	7
15	1
16	0

Small World Networks

Regular



Small World



https://slideplayer.com/slide/6262377/21/images/22/D.+J.+Watts+and+Steven+Strogatz+%28June+1998%29.jpg

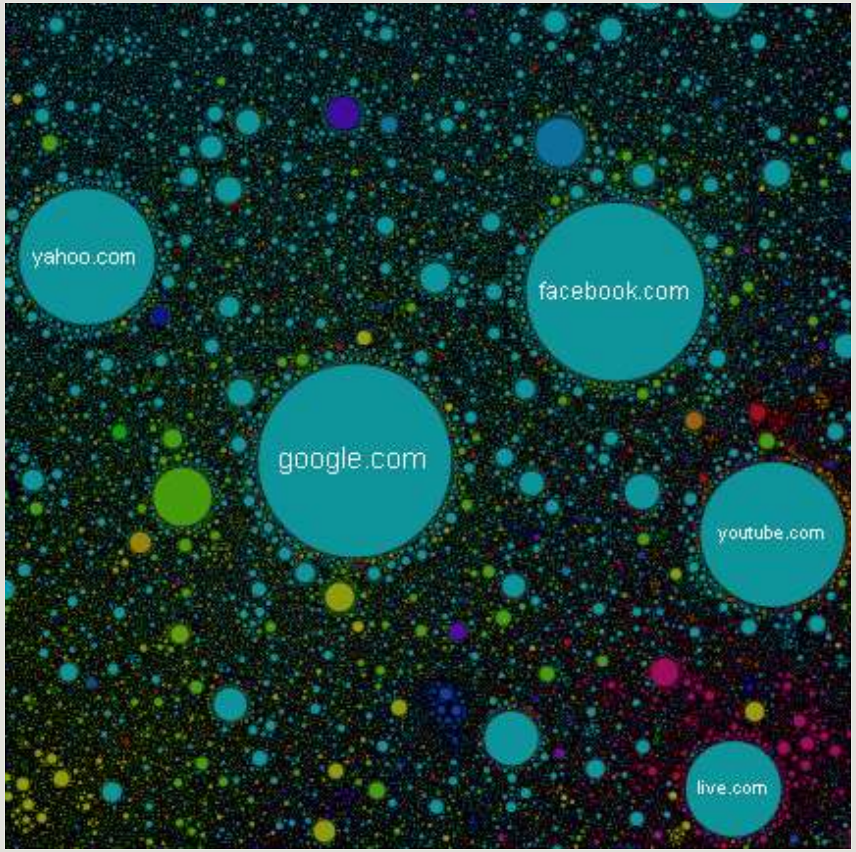
Scale Free Networks

Scale Free Networks

"Sprinkled among every walk of live… are a handful of people with a truly extra ordinary knack of making friends and acquaintances. They are connectors"

Malcom Gladwell

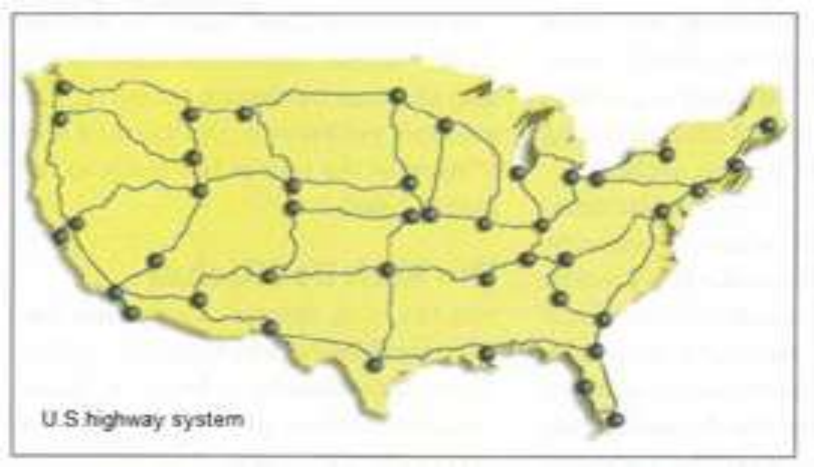
Scale Free Networks

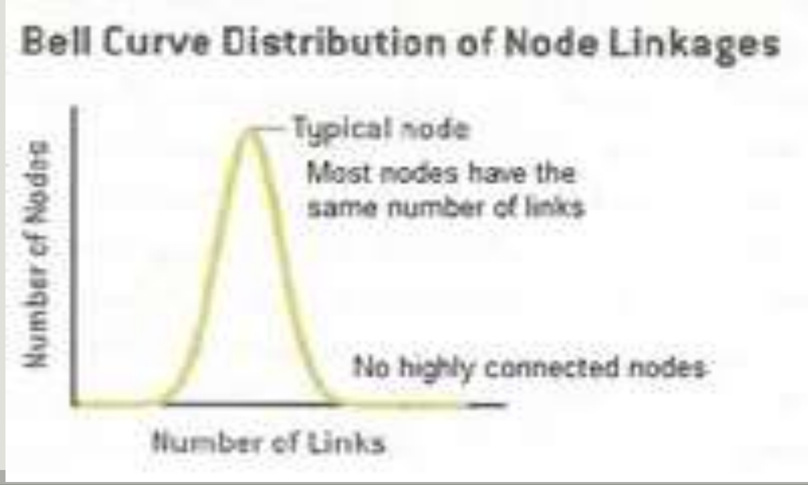


https://tekeye.uk/computing/how-many-websites-are-there

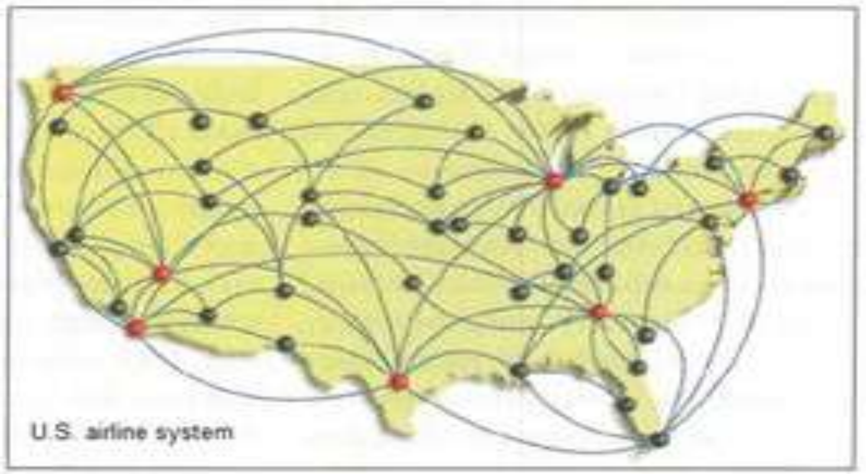
Hubs & Connectors

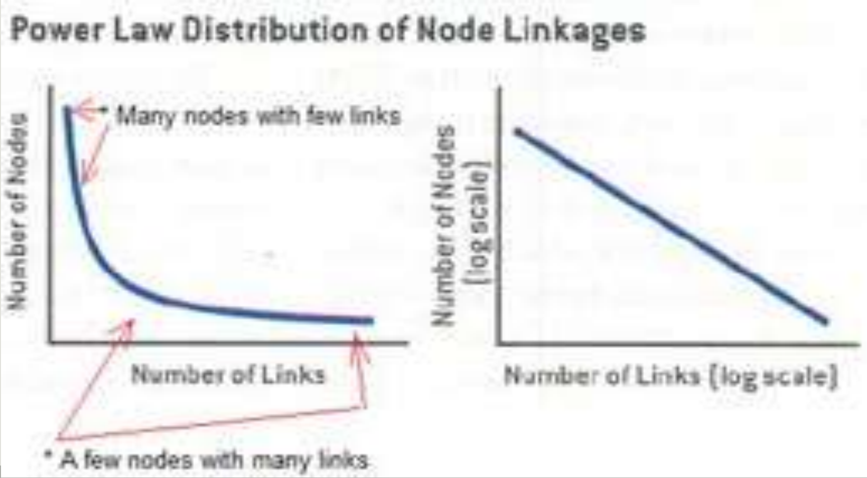
Random Networks





Scale Free Networks



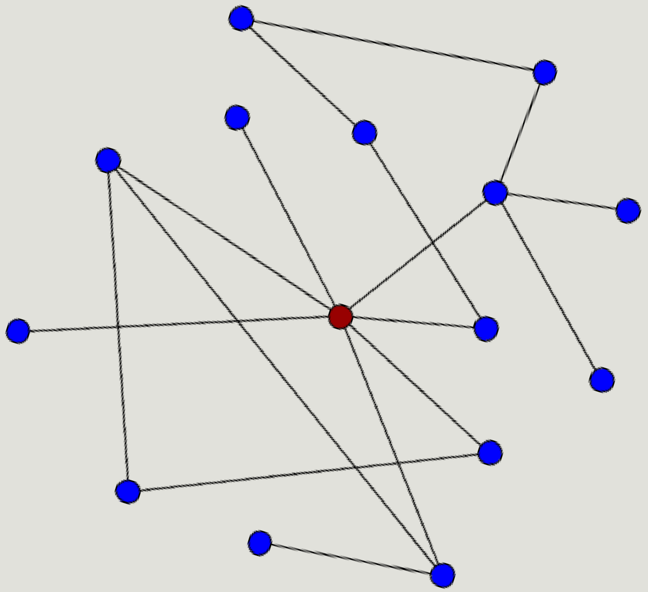


https://slideplayer.com/slide/9359https://slideplayer.com/slide/9359558/558/28/images/2/Random+Networks+Vs.+Scale+Free+Networks.jpg

Scale Free Networks

Degree Distribution can be described as

$$P_{deg}(k) \propto k^{-\gamma}$$



https://mathinsight.org/scale_free_network

Degree Distribution is the probability that a randomly chosen node has *k* connections

Scale Free Network

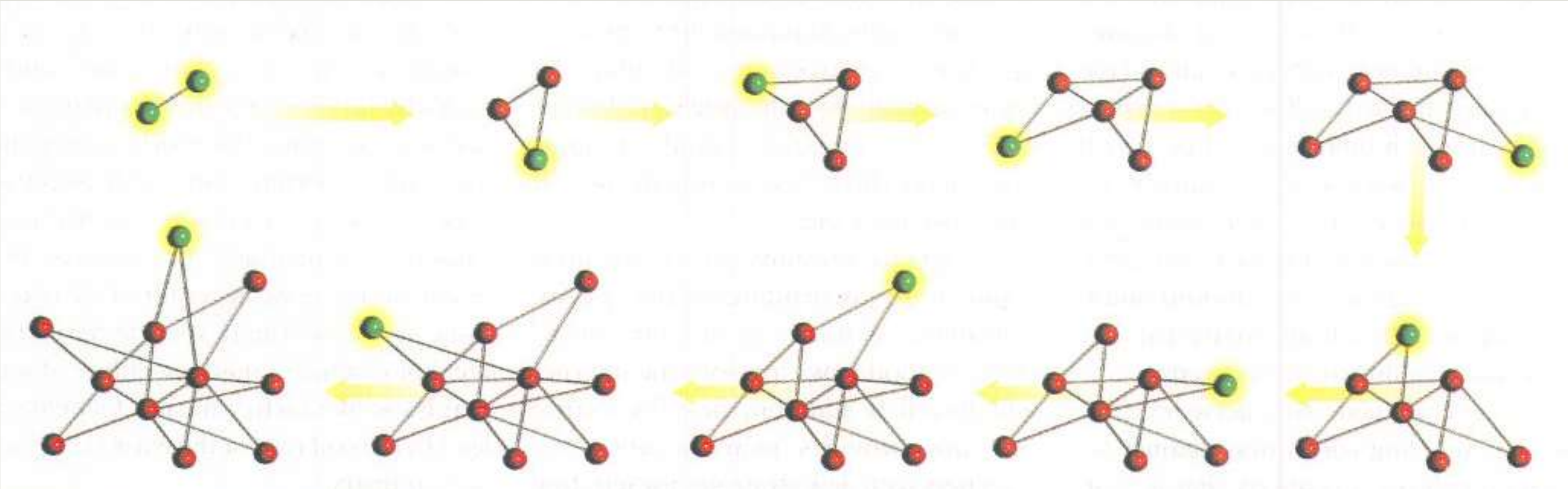
Growth

- Each network starts with one node and than grows by adding new ones.

Preferential Attachment

- The new nodes prefer to link to the nodes with more links.

Scale Free Networks



Scale-Free Networks by Albert-Laszlo Barabasi & Eric Bonabeau, Scientific American, 2003

Scale Free Networks

Examples of Scale-Free Networks

NETWORK	NODES	LINKS
Cellular metabolism	Molecules involved in burning food for energy	Participation in the same biochemical reaction
Hollywood	Actors	Appearance in the same movie
Internet	Routers	Optical and other physical connections
Protein regulatory network	Proteins that help to regulate a cell's activities	Interactions among proteins
Research collaborations	Scientists	Co-authorship of papers
Sexual relationships	People	Sexual contact
World Wide Web	Web pages	URLs

Scale-Free Networks by Albert-Laszlo Barabasi & Eric Bonabeau, Scientific American, 2003

Outlook

https://cheezburger.com/7940987904