The strategy game Footsteps

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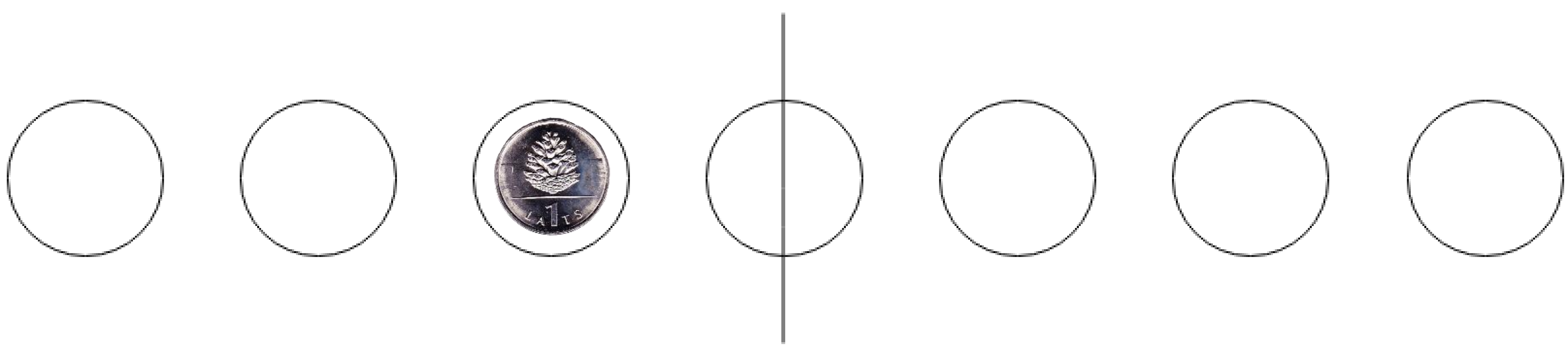
Agenda

1 Introduction

- Game theory developed over time
  - Was also applied in evolutionary biology
- Footsteps is also known as „Quo Vadis“
- One of many examples for strategy and decision-making
- Not as simple as it seems
  - Strategies can be a key role in a game of Footsteps

2 Footsteps
2.1 Basics and Rules

- Simple psychological game
- Requires:
  - Two players
  - Pencil
  - Paper
  - A token or coin
- 7 circles and the middle one separated by a line



2 Footsteps
2.1 Basics and Rules

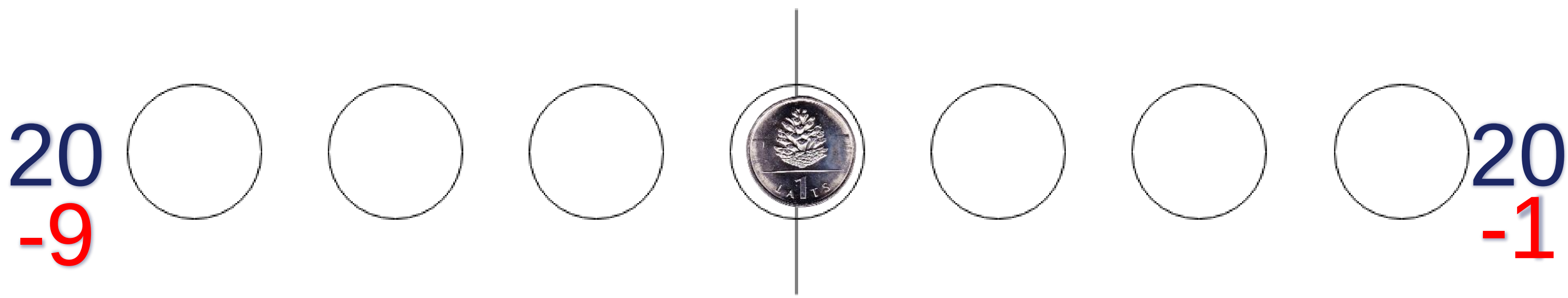
- each player starts with 50 points
- write down a number between 1 and their remaining points
- Larger number wins

  →Player can move token
- Objective is to reach the last circle of your opponents territory
- you can easily change the difficulty by adjusting the board size or amount of points

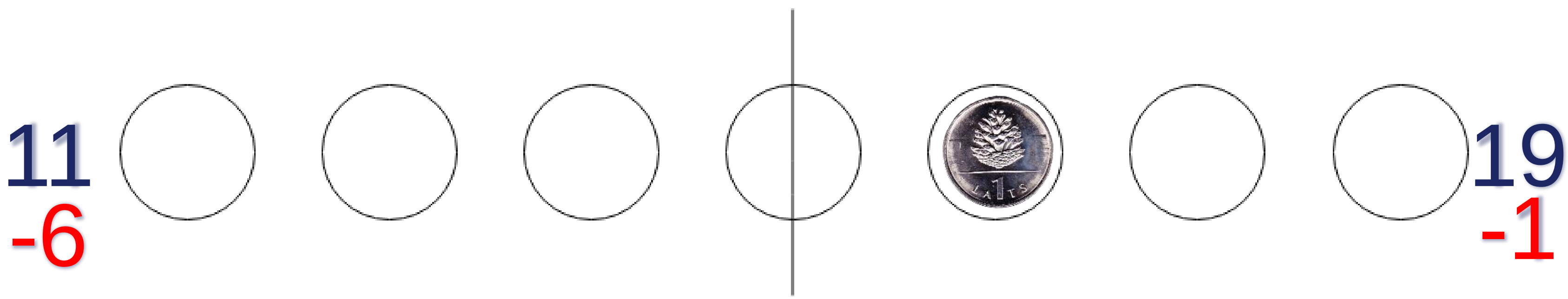
2.2 Game example

- Wining every turn with only one point would be ideal
  - Leads to a significant advantage
- Sometimes reasonable to face a certain loss
- Know how many points your opponent has to play accordingly
  - e.g. one player has 5 points left and the other none

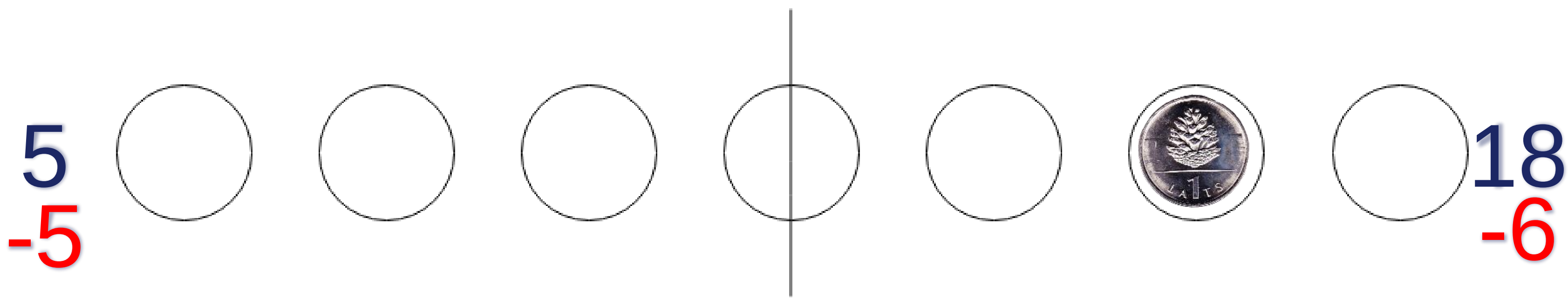
2 Footsteps
2.2 Game example



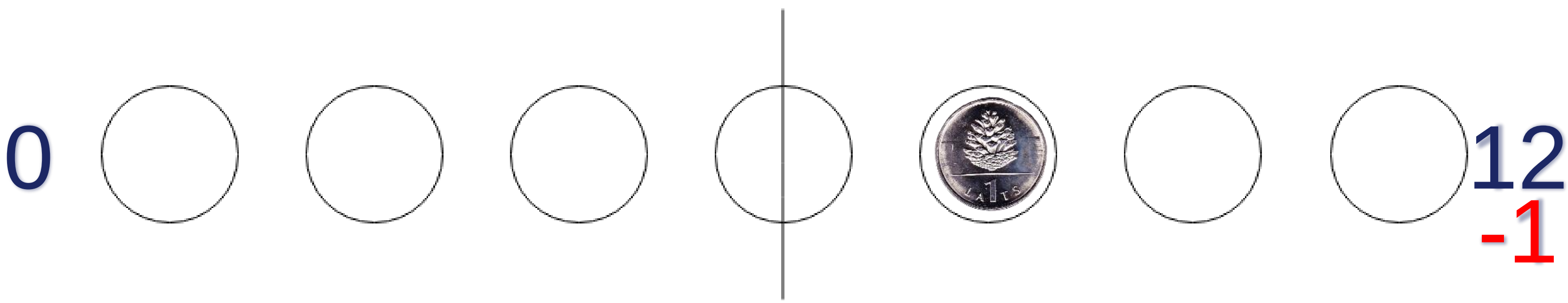
2 Footsteps
2.2 Game example



2 Footsteps
2.2 Game example



2 Footsteps
2.2 Game example



2 Footsteps
2.2 Game example

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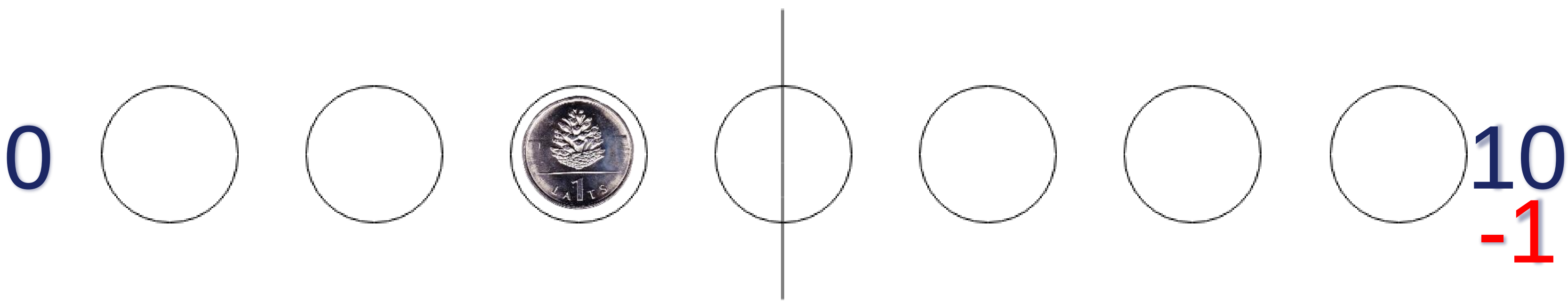
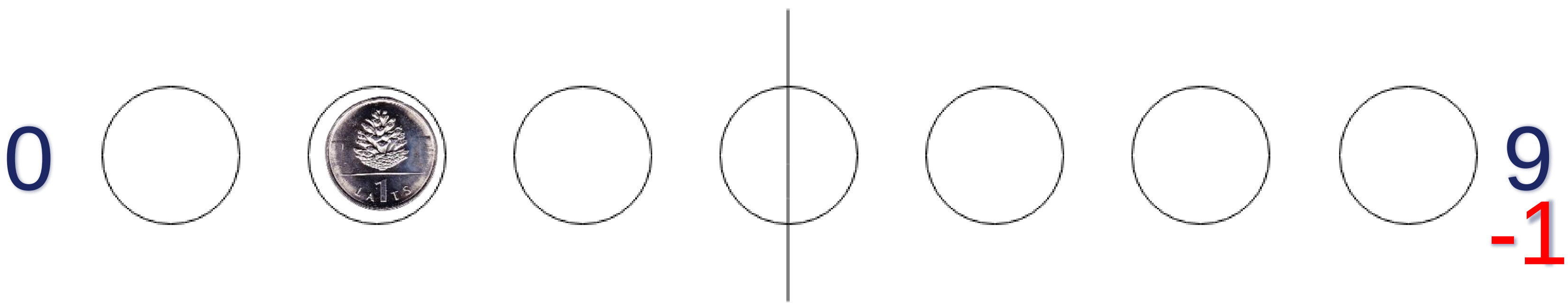
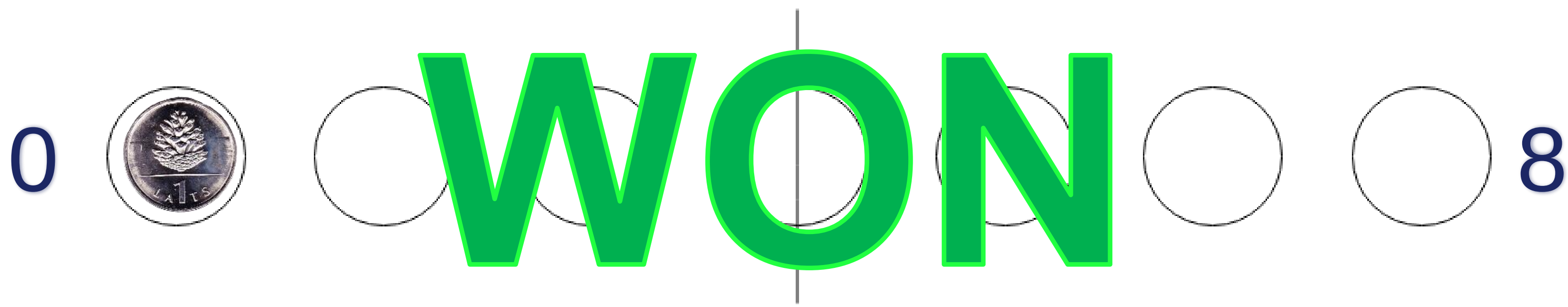


2 Footsteps
2.2 Game example

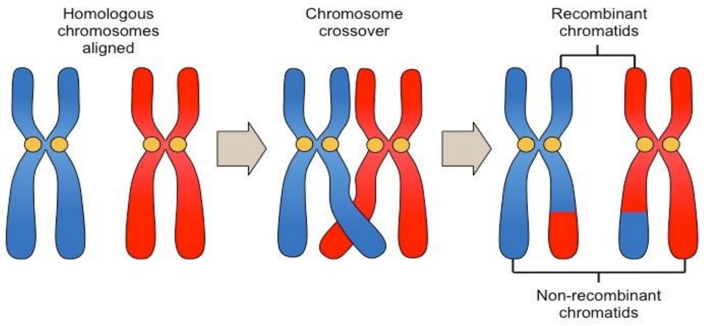


2 Footsteps
2.2 Game example



2 Footsteps
2.3 Strategies

- Strategically quite complex
  - Whether attack or counterattack
- Deterministic and probabilistic strategies to find out the best way of playing Footsteps
  - Using genetic algorithm (crossing over, mutation and selection)



- Represent a game by a collection of game states
  - 3 values each
  - 1st remaining points player 1
  - 2nd remaining points player 2
  - 3rd position of the token (1-7)

    → 50*50*7 = 17.500 mostly possible game states

- For analysis this representation was scaled down
  - Only 20 points in the beginning and 5 circles

    → 20*20* 5 = 2.000 states

    - Preserving the key aspect of game-play and is more receptive for selective pressure
  - Grouping the circles into 3 groups

    → 20 * 20 * 3 = 1.200 states

    - e.g. [20,20,3] for the initial position

2 Footsteps
2.3.1 The Deterministic Strategies

- Deterministic → every gene on a chromosome instructs how many points to spend
- Every gene on the chromosome coincide to a unique game state
- The value of the gene being the amount of points
  - For example gene [21,14,3] could be (1 … 21)
- As far as every gene has a value the individual is able to play against every opponent

2.3.2 The Probabilistic Strategies

- Extension of deterministic strategies
- Each gene consist of the same information
  - Structured [p, q, c] like before
- The value of each genes is a tuple of ‘p’ values
  - e.g. [4,9,3] = {30,60,70,40}
  - $\mathbf{1}: \frac{30}{30+60+70+40} = \frac{3}{20}$ $\mathbf{3}: \frac{70}{30+60+70+40} = \frac{7}{20}$
  - $\mathbf{2}: \frac{60}{30+60+70+40} = \frac{6}{20}$ $\mathbf{4}: \frac{40}{30+60+70+40} = \frac{4}{20}$
- As far as every gene has a value the individual is able to play against every opponent

2 Footsteps
2.3 Strategies

- Both strategies were evolved separately
- Strategies competed in an ‘round-robin-league, to decide about each individuals fitness
  - Winner gets 3 points, a drawn game results in 1 point each
- Uniform crossover was about 60%
- Probability of mutation was $\frac{2}{3N}$, where N is the number of genes
  - every three chromosomes should experience two mutations between them

2 Footsteps
2.3.3 The Deterministic Result

[0.4]	[0.5]	[0.7]	[1.2]	[1.1]	[1.2]	[1.6]	[1.8]	[2.3]	1.4	3.1	2.1	3.4	2.6	3	4.1	4.3	5.2	2.5
[0.5]	[0.7]	[1.3]	[0.5]	[1.0]	[1.9]	[2.1]	[1.5]	[1.5]	2.4	1.2	2	3.2	3.5	4.4	2.1	5.8	2.1	6.2
[0.5]	[0.6]	[0.8]	[1.2]	[1.2]	[1.4]	[2.0]	[1.4]	[2.2]	[3.6]	2.3	2.1	3.8	2.2	5.1	4.6	2.8	2.8	2.4
[0.4]	[0.6]	[0.5]	[1.4]	[0.5]	[0.7]	[2.1]	[2.0]	[1.5]	[2.0]	1.3	3.5	3.5	2.7	1.8	4.1	3.4	5.3	5.3
[0.1]	[0.4]	[0.9]	[1.7]	[0.5]	[1.5]	[1.2]	[1.9]	[2.5]	2.3	3.7	1.2	[0.8]	2.4	3.2	4	1.2	2.6	5.9
[0.5]	[0.8]	[1.0]	[1.2]	[1.9]	[1.2]	1.1	1.1	[2.4]	2.9	[4.2]	3	3.3	2.1	3.1	3	3.9	5.9	5.1
[0.5]	[0.5]	[1.1]	[0.8]	[1.5]	[1.7]	[0.8]	[3.1]	1.6	[1.9]	2.3	[2.7]	[3.2]	4.3	3.2	2.9	2.2	3.4	5.5
[0.4]	[0.1]	[0.7]	[1.5]	[1.8]	[1.5]	[2.5]	[2.6]	[1.7]	[1.4]	1.1	[2.7]	[1.8]	1.5	1.1	3.9	3.5	2.7	4.2
[0.1]	[0.8]	[0.9]	1	[2.2]	[1.2]	[0.8]	[1.8]	[3.1]	1.8	4.4	2.8	3.2	3.3	4.1	4.7	2.3	4.4	5.8
[0.5]	[0.3]	[0.7]	0.8	[1.8]	[1.4]	[1.8]	[3.0]	2.1	2.8	3.7	0.6	2.6	3.2	3.8	3.9	5.4	4.5	5.3
[0.1]	[0.5]	[0.9]	[1.5]	1.4	[2.0]	[1.8]	1.7	[1.4]	[2.3]	3.2	3.5	5.2	2.2	4.6	4.2	3.3	7.2	2.6
[0.2]	[0.7]	1.1	1.1	[1.4]	[2.3]	1.6	2.5	[1.5]	[1.5]	2.4	4.6	2.6	5.1	3	3	3.8	3.8	5.9
0.2	0.7	0.9	[0.9]	1.2	1.2	2	2.3	2.2	1.7	[3.6]	2.7	2.1	3.6	2.5	5.3	4.4	4	1.8
0.3	0.4	0.9	1.1	[2.0]	[0.9]	2	0.6	2.9	3.6	2.8	[1.9]	4.6	2.1	1.1	4.9	5.3	6.6	5.4
0.2	0.6	[0.7]	1.1	[2.1]	[2.2]	0.8	1.2	2	2.8	2.8	2.9	3.9	3.1	2.5	3.6	5.3	3.6	3.8
0.4	0.8	1	0.9	0.7	1.1	1.8	1.6	1.8	2.7	4	3.2	3.7	5.2	3.1	4.6	4.1	4.7	3.4
0.3	0.4	1	0.9	1.2	1.8	2.4	2.4	2.3	2.8	3.1	1.5	2.4	2.7	[3.7]	4.2	3.7	3.1	3.5
0.4	0.6	1	0.9	0.8	1.5	2.8	1.9	1.9	3.6	1.9	2.4	2.7	4	3.2	[3.1]	2.5	3.4	2.4
0.4	0.5	0.6	0.9	1.5	0.8	1.9	1.8	2.4	3.7	4.5	3.4	2.5	1.8	4	2	3.1	2.7	6.2
0.5	0.2	1	1.5	1.5	1.4	1.6	2.3	3.1	3.2	2.8	3.1	3.5	4.2	5	3.2	3.8	4.1	4
0.5	0.5	0.8	0.6	1	1.5	1.7	2.6	1.9	1.1	0.8	4.2	2.3	1.6	1.6	3.2	4.2	2.3	[4.2]

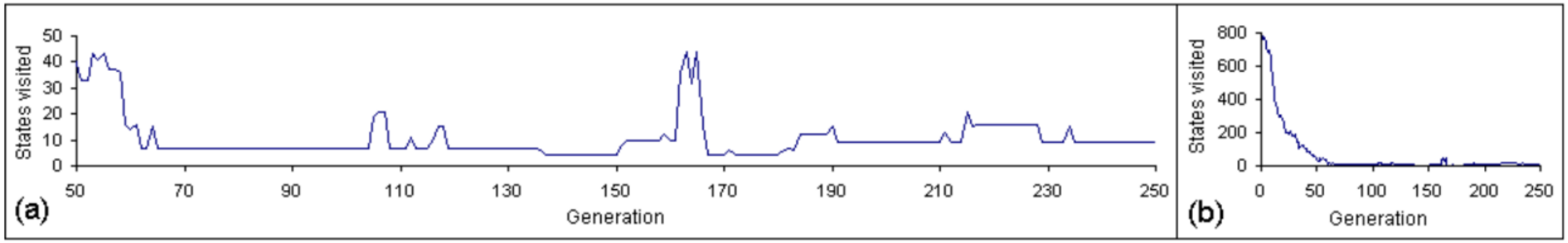
0	0	0.5	0	0	1	0	0	0.4	0	3.5	0	0	0.7	0	0	0	0	0
0	0	0	0	0.9	0	0	0	0.7	0.3	0.5	0	3.8	0	0	0	1	0	0
[0]	0	0	0	0.2	1.5	0.2	0	0.1	0.6	0.5	0.1	0	0	4.8	3	0	1.5	0
0	0	0.2	0	0	0	0	1.3	1.4	0.2	0.5	1	0	0	0	0	0	0.2	0.4
0	0.1	0	0	0	0	1.8	0	2.7	0	5	0.4	0	0.1	1.4	0	0	0.4	0
0	0	0	[0]	0	0	0.3	0	0	0	0	0	0.5	1.5	0	0	0.2	0	0.3
0	0	0.2	0.1	1.4	0.2	0	0.7	0	0	0	0	2.4	0.3	0	1.1	0	0	0
0	0	0	0.2	0	0.3	0	0	0.1	0	0	1.2	0	0	0	0.3	0.3	0	0
0	0	0	0.2	2.4	0.8	0	0	0	0	0	0	0.5	0.5	0.4	6	1.9	2.5	0.5
0	0.1	0	0	1.2	0	0	0	0	0.1	0	0	0	0	0	0	0	0.4	0
0	0	0	0	0	0	1	1	0	1.7	0	0	0	0	0	0	0	0	2.6
0.2	0.1	1	0	0	0	0.2	0	0	[0]	1.9	0	0	0	0	0	0.4	0	0
0	0	0.5	0	0.3	0	0	0	0.4	0	1.4	0.1	0.8	1.8	0.9	0	0	0	1.2
0	0	0	0	0	0	2	0	0	0	0	0	0.3	0	0.3	0	0	0	0
0	0	0	0	0	2.3	1.1	0	0	1.5	0	0	0	0	0	0	0	2	0
0.2	0.2	0	0.9	0	0.8	1.5	0.1	0	0	0	0.9	0.6	0	0	0	0	0	1.7
0	0	0	0	0	0	0	1.5	0.9	1.3	0	0.9	2.5	0.3	3	0	0	0	0
0	0	1.5	0	0	0	3	0.4	2.5	0.1	0	2.9	3.2	1.1	0	0	4.1	0.8	0
0	0	0.5	0.4	0	0	0	0	0.5	2.9	0	0.1	0.3	1.5	3.9	0.1	0	0.3	0
0	0	1	0	0.5	0.4	0.7	2.4	0.2	0	0.1	0	0.8	0	0.8	0.2	0.4	0	0.9
0	0	0	0	0	0.5	0	0	0.8	0	0	0	0	0	0	1	0	0.5	[0]

- Standard deviations of the gene values at state [*,*,3] = centered token
- Squared brackets → state visited at least one time
- Left figure after 20 generations / right figure after 200 generations
- Every game in the 200th generation progressed the same way
  - ([20, 20, 3], [11, 11, 3], [5, 5, 3], [2, 2, 3], [1, 1, 3], [0, 0, 3] → game tied

2 Footsteps
2.3.3 The Deterministic Result

- About 2/3 of the standard deviations are 0 in generation 200
  - Individuals are close to convergence
- Every game itself is a phenotype
- The population converged in the four important game states, but not in the others
  - Population was unconverged genotypically, but converged phenotypically

2 Footsteps
2.3.3 The Deterministic Result

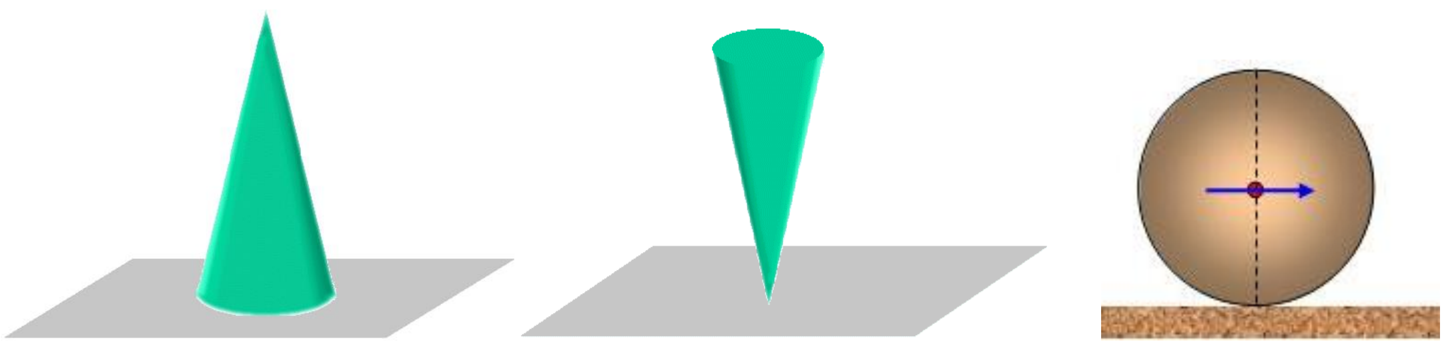


- Graphs show how evolution progressed
- Plateaus indicate, that phenotypes were mostly identical
  - Doesn't mean there was no mutation

  → There were no genotypical changes which changed the phenotypical appearance
- the evolution is a result of the foregoing experiments
  - In genetic algorithms the environment is a key influence
    - Population == Environment
  - Mutations change the environment and introduce new potential to other individuals
- The fitness of one individual depends wholly on the others alongside

2 Footsteps
2.3.3 The Deterministic Result

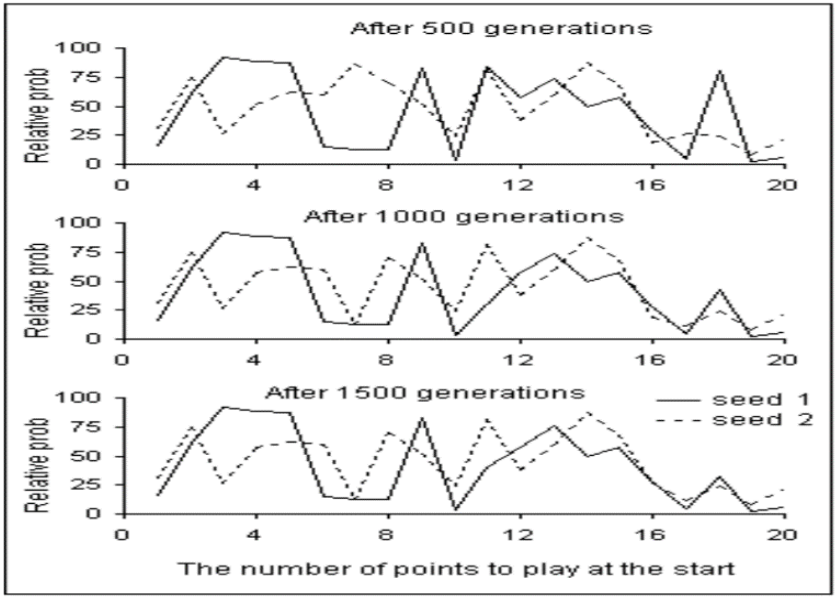
- In case of Footsteps a deterministic strategy is either ‘good or ‘bad
  - Outcome depends on the given opponents
- ‘Footsteps-Individuals’ are not evolving in traditional manner towards a good solution
  - They enter a state similar the neutral equilibrium



Stable Equilibrium | Unstable Equilibrium | Neutral Equilibrium

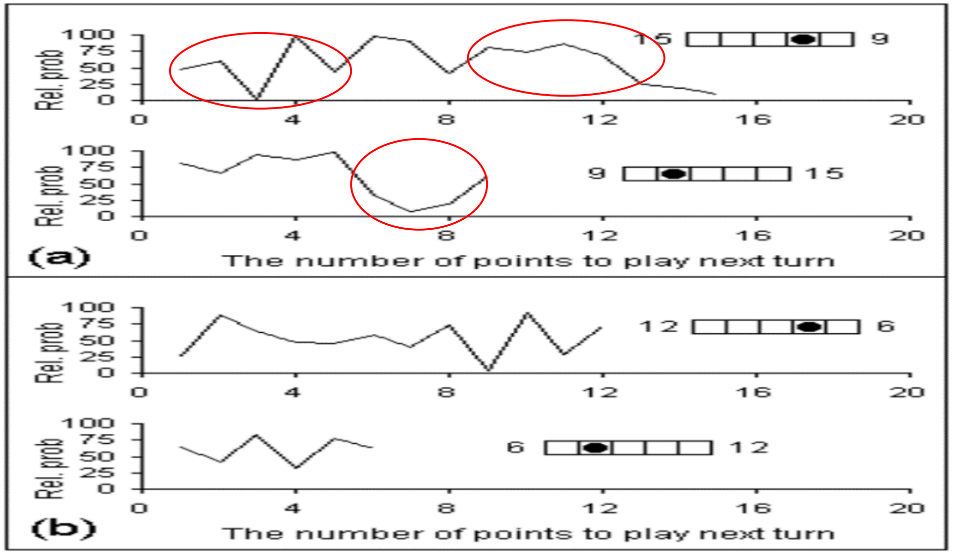
2 Footsteps
2.3.4 The Probabilistic Result

- Probability of spending a certain amount of points at the beginning
- The distributions belong to generations 500,1000 and 1500
- Similarities indicate that the populations were nearly converged
- 2 interesting facts
  - Especially in seed 2 the probability to spend a high amount of points on the first turn is low compared to others
  - Among the other values there is no dominant one
    - Wide range of choices for the individual
    - →There is no 'best-choice'
      - →***unpredictability***



2 Footsteps
2.3.4 The Probabilistic Result

- The both top plots show a similar situation as in the previous plot
  - Not to many points and across the other choices unpredictability
- The bottom plot of (a) represents the fear of failing to win, more than the fear of losing in that particular turn
  - Low possibilities between 5 – 9
- Figure (b) looks quite like (a) but is much noisier
  - Indicating a low mutation rate



2 Footsteps
2.3.4 The Probabilistic Result

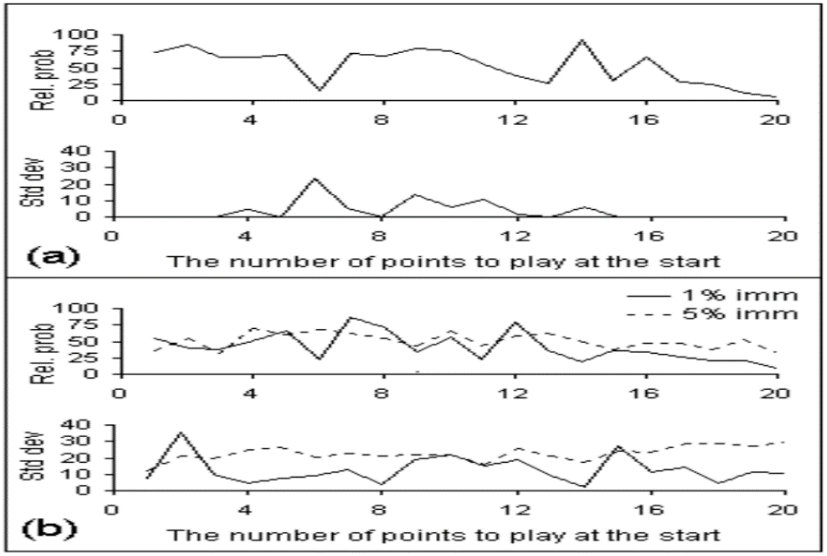
- Very few genes in the chromosome contribute to the phenotype
  - Selective pressure is only sparely applied
- Genes can be mutated negatively without consequences
- A non-low mutation rate can be problematic
  - ‘good’ evolutionary changes will probably be corrupted while not expressed
- A low mutation rate leads to little exploration
- Hence a difficult trade-off arises:

$$safe\ building\ blocks \,\&\, insufficient\ exploration$$
$$vs.$$
$$sufficient\ exploration \,\&\, unsafe\ blocks$$

2 Footsteps
2.3.4 The Probabilistic Result

- Outcome of an evolution with too much mutation
- (a):
  - Desired average genotype (top plot)
  - Quite high standard deviations across the population (bottom plot)

    → high variety
  - High mutation rate did not lead to a precise genotype
- (b):
  - Trying to bypass the problem of (a)
  - Use random immigrants
  - Effective raising of the standard deviations



3 Conclusion

- In genetic algorithms it is important to measure convergence phenotypically
  - Only some of the chromosome's genes contribute to their individual's phenotypical development.
  - Populations may converge phenotypically but not genotypically

- Deterministic Strategies are faulty, accounted by their exploitability
  - If you know what your opponent is going to do you can easily react to his turn
- Probabilistic Strategies work better
  - Situation of attack and counterattack
  - **Best approach is unpredictability (including chicanery and randomness)**

Thank you for your attention !

- Selected Sources:

  - Richard Dawkins. The Blind Watchmaker. Longman, first edition, 1986.
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