UNIVERSITY OF APPLIED SCIENCES

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# The strategy game Footsteps

*Maurice Behm (winf102354)*

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# Contents

# Introduction

The game Footsteps, also known as “Quo Vadis”, is only one of the examples which allow us to get in contact with the mathematical model of game-theory. There are a lot of prominent examples such as the prisoner’s dilemma, which offer more possibilities, in the sense of strategy and decision-making, than you might imagine at first glance.

But why should you consider getting in contact with game-theory and invest time into understanding how to play a game if you could just enjoy it? First of all, it is important to know that game-theory does not only include games. It started in the 1930s as a branch of applied math and developed over time. It was applied in many cases throughout the years, for example as an asset during World War II and in evolutionary biology.

Today’s game-theory often applies to political, management and network problems. But there are also some other examples from the real world which should motivate you to take a closer look at this topic. For example, in the early 2000s Arsène Wenger, a former Arsenal FC coach, was known for his fast counterattacking football and because of that the opponent’s coach, in this case, Graham Taylor of Aston Villa, decided to play as defensively as possible against Arsenal.

Because of that decision, Villa was able to keep the 0:0 and the associated point instead of losing. Wenger criticized Taylor’s attitude, but overall this was simple game-theory.

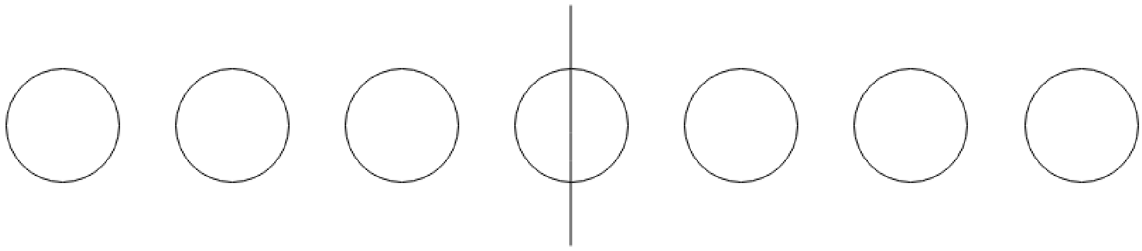
Why especially this example perfectly fits to the strategies we are going to discuss later on, is because it behaves similarly to a game of Footsteps. Strategies are a big topic and key component of every game theory, they can be developed in many different ways and lead to unpredictable results depending on the situation.

# Footsteps

## Basics and Rules

Footsteps is a short and simple psychological game which only requires two players, pencil and paper. In a short amount of time footsteps shows how complex it really can be. It combines mind-reading, mental arithmetic and a little bit of luck.

All you need to play the game is a paper with seven circles on it, each of them big enough to fit a small token or coin inside of it. The middle circle is separated by a single line. So, in the end, there are three circles for each player and one circle which belongs to both players as seen below.



In the beginning, each player starts with 50 points to spend them on their turns. On each turn, both players write down a number between 1 and their remaining points that they want to spend in this round.

The numbers are compared and the player with the larger number can move the token, which is placed in the center circle in the beginning of the game, into the opponent's direction to conquer their territory.  If the chosen number of both players is equal the token is not moved and the game continues. The objective of the game is to reach the last circle of your opponent's territory or in other words to win three more turns than the other contestant.

If each player used up their points, it is up to the players to decide whether agree on a draw or a half-victory for the one who achieved to get the token out of their territory.

That is basically the game, but there is definitely more to it than it seems. Not only you can easily change the difficulty by adjusting the board size or amount of points, but there are also a lot of strategic aspects to this game.

## Game example

First of all, it is pretty obvious, that the ideal gameplay is if you win every turn with only one point. So, you are getting the token in your opponent half without spending a significant amount of points more than him. Even if you lose one or two turns this could give you an important advantage against the other player.

This concludes to the next interesting aspect of Footsteps. Sometimes it is a reasonable approach to face a certain loss, in this way you possibly get into the position, where you only have to know how many points the opponent has left and play accordingly. An extreme example could be that you have left 5 points and your opponent none.

In that case, even if you are one circle away from losing you will win because you only have to spend one point each turn.

But to understand the nicety of Footsteps it is helpful to go through a short game, where strategy plays an important role.

Imagine the token is inside the center circle and each player have spent 30 points, that is pretty much a variant of the game with 20 starting points. Now you consider allocating nine points and your opponent only the minimum of one point.
This situation leads to a game state where you are only two circles away from winning and 11 remaining points while the opponent is seemingly losing but with a remaining total of 19 points. On the next turn, for example, you play six points and the opponent again only one. That puts you in the circle which is only one turn away from winning, but the problem now is that there are only five points left on your bank and 18 on the other player's.
The only thing the opponent now has to do is to play 5 points in case you gamble with all your remaining points. The following turns are pretty obvious, with the outcome of your opponent winning.

This is only a short example, which shows how mind-reading and strategic-thinking plays an important role in this, at first glance, simple game.

## Strategies

Though Footsteps is simple in conception the game strategically is quite complex and clarifies the tactical dilemma of whether to attack or wait until the time has come for the counterattack.

In a paper of Robert Morris and Tim Watson, deterministic and probabilistic strategies were evolved. They make use of the genetic algorithm, which is a metaheuristic inspired by the process of natural selection and relies on bio-inspired operators such as mutation, crossover and selection. In their paper they concluded that unpredictability is the key to success.

A game of Footsteps can simply be represented by a collection of possible game states with 3 values each. The first value represents the remaining points of player one (0-50), the second the other player's points and the third value the position in which the token currently resides (1-7).

Thus, there are $50 * 50 * 7 = 17.500$ states in a game of Footsteps and most of them are feasible ones (e.g. the state [50,50,3] is not possible because the token would be moved without any points spent).

This representation of game states helps us to understand Footsteps as a journey through these states, always starting with the initial state [50,50,4]. Because the collection of game states when investigating a normal round of Footsteps is huge Morris and Watson scaled the game down to $20 * 20 * 5 = 2.000$ states.

This version was easier to analyze and more receptive for selective pressure though big enough to preserve the key aspects of game-play. Also, they grouped the circles (The 3 represents the middle 3 circles and the 1 and 2 the outer circles which imply the end of the game) so there is only a total of $20 * 20 * 3 = 1.200$ genes to look at for each chromosome.

## The Deterministic Strategies

For this strategic approach, every gene on the chromosome coincide to a unique game state, with the value of the gene being the information of how many points to spend in that current state.

This strategy is called deterministic because for every gene (game state) on a chromosome there is a value, that instructs how many points to spend. For example, the gene [21,14,3] contains an integer in [1...21], because that are the possible amounts the player could spend on its turn.

In conclusion, if every gene in a given individual's chromosome is set to a viable value, this individual is able to play against any opponent, because for every game state that can arise, there is an instruction of what it has to do.

## The Probabilistic Strategies

These Strategies are an extension of the deterministic ones. This time an individual's chromosome consists of a gene for each state (p, q, c), again representing the same information as before. But the value of each gene has "p" genes on its own, varying from 1 to 99, representing the relative probability of that number of points being spent. This results in a chromosome length of 13.167 genes [2].
To get a better understanding of how these genes work, here is an example.
The state [4,9,3] has a value of {30,60,70,40}, that means that the individual, when in this situation, would have the following tendencies to spend the different amounts of points:

$$\mathbf{1}\text{:}\ \frac{30}{30+60+70+40}=\frac{3}{20} \quad \mathbf{3}\text{:}\ \frac{70}{30+60+70+40}=\frac{7}{20}$$

$$\mathbf{2}\text{:}\ \frac{60}{30+60+70+40}=\frac{6}{20} \quad \mathbf{4}\text{:}\ \frac{40}{30+60+70+40}=\frac{4}{20}$$

Like it was the case at the deterministic strategies, the individual is able to play against any opponent when every gene in its chromosome is set to a viable value.

Afterwards both strategies were evolved separately, because the genes weren't able to improve without changing the parameters. To keep the simplicity Morris and Watson decided not to extend the parameters. To decide about the fitness of each individual, both types of strategies competed in a 'round-robin-league', where a winner gets 3 points and drawn game leads to one point each. These points are the fitness.

For the evolution, standard proportionate selection was used (The understanding of this theory is too prescriptive for this assignment, but yet very interesting). The uniform crossover was about 60% on average, which means that 60% of the new gene is a recreation of the two parents, and the mutation rate was set to a value where every three chromosomes should experience two mutations between them, so the genewise mutation probability was $\frac{2}{3N}$ , where $N$ is the number of genes in a chromosome.

## Deterministic Result

The following figures show the extent of the convergence of population pertaining to the deterministic strategies. In particular, they illustrate the standard deviations of the gene values at the game state [*, *, 3], which is the state with a centered token (which makes up $\frac{1}{3}$).

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [0.4] | [0.5] | [0.7] | [1.2] | [1.1] | [1.2] | [1.6] | [1.8] | [2.3] | 1.4 | 3.1 | 2.1 | 3.4 | 2.6 | 3 | 4.1 | 4.3 | 5.2 | 2.5 |
| [0.5] | [0.7] | [1.3] | [0.5] | [1.0] | [1.9] | [2.1] | [1.5] | [1.5] | 2.4 | 1.2 | 2 | 3.2 | 3.5 | 4.4 | 2.1 | 5.8 | 2.1 | 6.2 |
| [0.5] | [0.6] | [0.8] | [1.2] | [1.2] | [1.4] | [2.0] | [1.4] | [2.2] | [3.6] | 2.3 | 2.1 | 3.8 | 2.2 | 5.1 | 4.6 | 2.8 | 2.8 | 2.4 |
| [0.4] | [0.6] | [0.5] | [1.4] | [0.5] | [0.7] | [2.1] | [2.0] | [1.5] | [2.0] | 1.3 | 3.5 | 3.5 | 2.7 | 1.8 | 4.1 | 3.4 | 5.3 | 5.3 |
| [0.1] | [0.4] | [0.9] | [1.7] | [0.5] | [1.5] | [1.2] | [1.9] | [2.5] | 2.3 | 3.7 | 1.2 | [0.8] | 2.4 | 3.2 | 4 | 1.2 | 2.6 | 5.9 |
| [0.5] | [0.8] | [1.0] | [1.2] | [1.9] | [1.2] | 1.1 | 1.1 | [2.4] | 2.9 | [4.2] | 3 | 3.3 | 2.1 | 3.1 | 3 | 3.9 | 5.9 | 5.1 |
| [0.5] | [0.5] | [1.1] | [0.8] | [1.5] | [1.7] | [0.8] | [3.1] | 1.6 | [1.9] | 2.3 | [2.7] | [3.2] | 4.3 | 3.2 | 2.9 | 2.2 | 3.4 | 5.5 |
| [0.4] | [0.1] | [0.7] | [1.5] | [1.8] | [1.5] | [2.5] | [2.6] | [1.7] | [1.4] | 1.1 | [2.7] | [1.8] | 1.5 | 1.1 | 3.9 | 3.5 | 2.7 | 4.2 |
| [0.1] | [0.8] | [0.9] | 1 | [2.2] | [1.2] | [0.8] | [1.8] | [3.1] | 1.8 | 4.4 | 2.8 | 3.2 | 3.3 | 4.1 | 4.7 | 2.3 | 4.4 | 5.8 |
| [0.5] | [0.3] | [0.7] | 0.8 | [1.8] | [1.4] | [1.8] | [3.0] | 2.1 | 2.8 | 3.7 | 0.6 | 2.6 | 3.2 | 3.8 | 3.9 | 5.4 | 4.5 | 5.3 |
| [0.1] | [0.5] | [0.9] | [1.5] | 1.4 | [2.0] | [1.8] | 1.7 | [1.4] | [2.3] | 3.2 | 3.5 | 5.2 | 2.2 | 4.6 | 4.2 | 3.3 | 7.2 | 2.6 |
| [0.2] | [0.7] | 1.1 | 1.1 | [1.4] | [2.3] | 1.6 | 2.5 | [1.5] | [1.5] | 2.4 | 4.6 | 2.6 | 5.1 | 3 | 3 | 3.8 | 3.8 | 5.9 |
| 0.2 | 0.7 | 0.9 | [0.9] | 1.2 | 1.2 | 2 | 2.3 | 2.2 | 1.7 | [3.6] | 2.7 | 2.1 | 3.6 | 2.5 | 5.3 | 4.4 | 4 | 1.8 |
| 0.3 | 0.4 | 0.9 | 1.1 | [2.0] | [0.9] | 2 | 0.6 | 2.9 | 3.6 | 2.8 | [1.9] | 4.6 | 2.1 | 1.1 | 4.9 | 5.3 | 6.6 | 5.4 |
| 0.2 | 0.6 | [0.7] | 1.1 | [2.1] | [2.2] | 0.8 | 1.2 | 2 | 2.8 | 2.8 | 2.9 | 3.9 | 3.1 | 2.5 | 3.6 | 5.3 | 3.6 | 3.8 |
| 0.4 | 0.8 | 1 | 0.9 | 0.7 | 1.1 | 1.8 | 1.6 | 1.8 | 2.7 | 4 | 3.2 | 3.7 | 5.2 | 3.1 | 4.6 | 4.1 | 4.7 | 3.4 |
| 0.3 | 0.4 | 1 | 0.9 | 1.2 | 1.8 | 2.4 | 2.4 | 2.3 | 2.8 | 3.1 | 1.5 | 2.4 | 2.7 | [3.7] | 4.2 | 3.7 | 3.1 | 3.5 |
| 0.4 | 0.6 | 1 | 0.9 | 0.8 | 1.5 | 2.8 | 1.9 | 1.9 | 3.6 | 1.9 | 2.4 | 2.7 | 4 | 3.2 | [3.1] | 2.5 | 3.4 | 2.4 |
| 0.4 | 0.5 | 0.6 | 0.9 | 1.5 | 0.8 | 1.9 | 1.8 | 2.4 | 3.7 | 4.5 | 3.4 | 2.5 | 1.8 | 4 | 2 | 3.1 | 2.7 | 6.2 |
| 0.5 | 0.2 | 1 | 1.5 | 1.5 | 1.4 | 1.6 | 2.3 | 3.1 | 3.2 | 2.8 | 3.1 | 3.5 | 4.2 | 5 | 3.2 | 3.8 | 4.1 | 4 |
| 0.5 | 0.5 | 0.8 | 0.6 | 1 | 1.5 | 1.7 | 2.6 | 1.9 | 1.1 | 0.8 | 4.2 | 2.3 | 1.6 | 1.6 | 3.2 | 4.2 | 2.3 | [4.2] |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.5 | 0 | 0 | 1 | 0 | 0 | 0.4 | 0 | 3.5 | 0 | 0 | 0.7 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.9 | 0 | 0 | 0 | 0.7 | 0.3 | 0.5 | 0 | 3.8 | 0 | 0 | 0 | 1 | 0 | 0 |
| **[0]** | 0 | 0 | 0 | 0.2 | 1.5 | 0.2 | 0 | 0.1 | 0.6 | 0.5 | 0.1 | 0 | 0 | 4.8 | 3 | 0 | 1.5 | 0 |
| 0 | 0 | 0.2 | 0 | 0 | 0 | 0 | 1.3 | 1.4 | 0.2 | 0.5 | 1 | 0 | 0 | 0 | 0 | 0 | 0.2 | 0.4 |
| 0 | 0.1 | 0 | 0 | 0 | 0 | 1.8 | 0 | 2.7 | 0 | 5 | 0.4 | 0 | 0.1 | 1.4 | 0 | 0 | 0.4 | 0 |
| 0 | 0 | 0 | **[0]** | 0 | 0 | 0.3 | 0 | 0 | 0 | 0 | 0 | 0.5 | 1.5 | 0 | 0 | 0.2 | 0 | 0.3 |
| 0 | 0 | 0.2 | 0.1 | 1.4 | 0.2 | 0 | 0.7 | 0 | 0 | 0 | 0 | 2.4 | 0.3 | 0 | 1.1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0.2 | 0 | 0.3 | 0 | 0 | 0.1 | 0 | 0 | 1.2 | 0 | 0 | 0 | 0.3 | 0.3 | 0 | 0 |
| 0 | 0 | 0 | 0.2 | 2.4 | 0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 | 0.4 | 6 | 1.9 | 2.5 | 0.5 |
| 0 | 0.1 | 0 | 0 | 1.2 | 0 | 0 | 0 | 0 | 0.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1.7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.6 |
| 0.2 | 0.1 | 1 | 0 | 0 | 0 | 0.2 | 0 | 0 | **[0]** | 1.9 | 0 | 0 | 0 | 0 | 0 | 0.4 | 0 | 0 |
| 0 | 0 | 0.5 | 0 | 0.3 | 0 | 0 | 0 | 0.4 | 0 | 1.4 | 0.1 | 0.8 | 1.8 | 0.9 | 0 | 0 | 0 | 1.2 |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0.3 | 0 | 0.3 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 2.3 | 1.1 | 0 | 0 | 1.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| 0.2 | 0.2 | 0 | 0.9 | 0 | 0.8 | 1.5 | 0.1 | 0 | 0 | 0 | 0.9 | 0.6 | 0 | 0 | 0 | 0 | 0 | 1.7 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.5 | 0.9 | 1.3 | 0 | 0.9 | 2.5 | 0.3 | 3 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1.5 | 0 | 0 | 0 | 3 | 0.4 | 2.5 | 0.1 | 0 | 2.9 | 3.2 | 1.1 | 0 | 0 | 4.1 | 0.8 | 0 |
| 0 | 0 | 0.5 | 0.4 | 0 | 0 | 0 | 0 | 0.5 | 2.9 | 0 | 0.1 | 0.3 | 1.5 | 3.9 | 0.1 | 0 | 0.3 | 0 |
| 0 | 0 | 1 | 0 | 0.5 | 0.4 | 0.7 | 2.4 | 0.2 | 0 | 0.1 | 0 | 0.8 | 0 | 0.8 | 0.2 | 0.4 | 0 | 0.9 |
| 0 | 0 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0.5 | **[0]** |

For example, the bottom-right value corresponds to the initial state [20, 20, 3]. A value in squared brackets indicates that this value/game state was visited at least one time during the league games. The left figure shows the standard deviations after a total of 20 generations indicating that the population is very genetically diverse. Two characteristics manifest this fact. First there are some big standard deviations across the figure showing that many individuals of the population acted differently. Secondly there are many squared brackets indicating, that the where many game states visited in the 20th generation.

The right figure displays the same type of information but this time after 200 genetical improvements. About $\frac{2}{3}$ of the standard deviations are zero, suggesting that the individuals in the 200$^{th}$ generation are close to convergence.

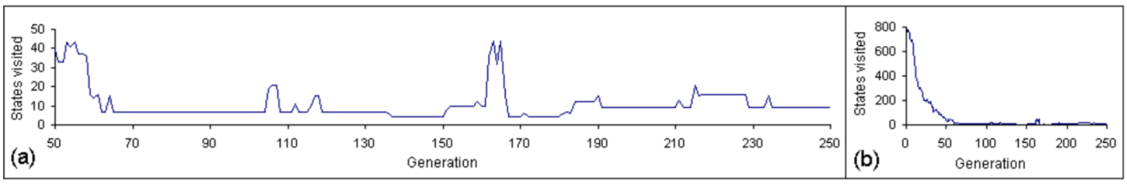
Furthermore, there are only four game states marked with squared brackets, showing that the population is converged in those loci.

Following therefrom is a pretty interesting fact because the population was unconverged genotypically, but it had converged phenotypically. That's the case, because the phenotypes in this case are the games themselves and every game in the 200$^{th}$ generation progressed in the same way ([20, 20, 3], [11, 11, 3], [5, 5, 3], [2, 2, 3], [1, 1, 3], [0, 0, 3] → game tied). This result was possible because the individual's chromosomes converged in the four important game states, but they did not in the others.

The next two graphs describe how the evolution progressed. They show how many game states were visited in each generation. Figure 'a' displays some plateaus, which means that the phenotypes of the included generations were mostly identical, also implying that there was no evolution until a rise terminates a plateau. This rise shows those incidents where a mutation happened. But that doesn't mean that there was no mutation between those generations included by the plateau. It only indicates that there were no genotypically changes which changed the phenotypical appearance of the individuals. In fact, a rise shows the occasion of a mutation which affects the essential game states.

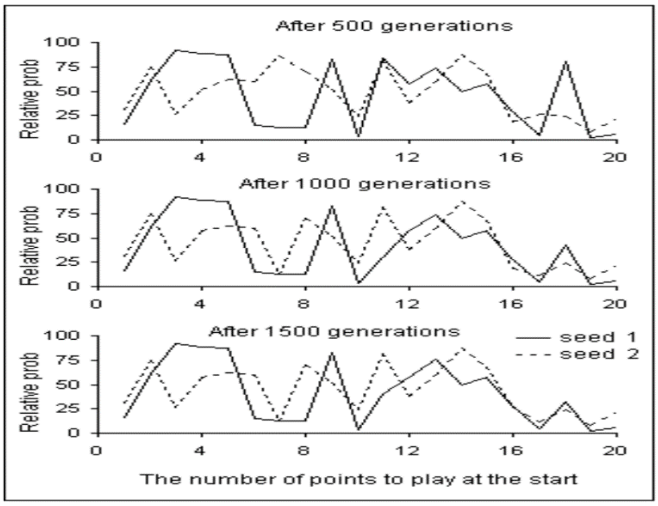
Important in this experiment is, that this development is a result of foregoing experiments. Because in strategy genetic algorithms like this the environment, which is the key influence for evolution, is the population itself. [1] Mutations therefore change the population/environment and introduce new optimization potential to other individuals. Some of these changes can be trivial or profound. [2] All in all, the fitness of one individual depends wholly on the other individuals alongside it.

In the case of Footsteps that means that a deterministic strategy is either 'good' or 'bad' depending on the given opponents. That means that the 'Footsteps-Individuals' are not evolving in a traditional manner, meaning that they evolve towards a 'good' solution, they enter a state pretty similar to the neutral equilibrium. Neutral equilibrium says that a body stays in the displaced position after it has been displaced slightly, in contrast to the stable and unstable equilibrium. They remain in this displaced position until they get shifted again, and so on. [3]



### Probabilistic Result

The first thing investigated was the probability of spending a certain amount of points at the beginning of the game. The below figure shows the relative probability, varying from 1 to 99, to play the associated amount at the start. The distributions belong to the whole population from the generations 500, 1000, and 1500. The similarities between these three snapshots indicate that the populations were nearly converged. Strengthening this thesis is the fact that the standard deviations were mostly zeroes(not shown in the plots). Even though these generations look nearly the same they are highly evolved populations and because of that there are two interesting things to notice. Firstly, especially in seed 2, the probability to spend a high amount of points on the first turn is low compared to the others, because that would make it nearly impossible to win. In other words:" Do not spend 16-20 points in the beginning". Secondly, you can notice that among the other values there is no dominant one, revealing that no 'best-choice' exists on the first turn. Additionally this shows, that there is a wide-range of choices for the individuals. In fact, this makes the player unpredictable at this point in the game.

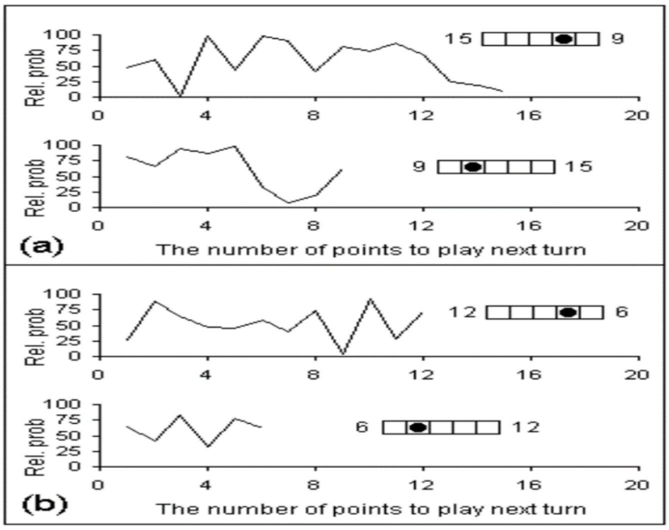



The next plots are the same pattern as in the figure before, but this time it describes two game states in which there is the possibility of ending the game. The top two graphs show the probability distribution for a player in the states [15, 9, 4] and the opposite [9, 15, 2].

In the first plot, there is a similar situation as in the figures above. The choices which make the game un-winnable have the lowest possibilities and across the other options, there is an approximately unpredictable choice.

The bottom plot of figure ‘a’, displays a similar situation, but the lower probabilities at the right end, representing the fear of failing to win, more than the fear of losing in that particular turn.

Figure ‘b’ took a similar pair of situations compared to ‘a’. Nevertheless, it is much noisier. The top graph is relatively uniform at the left and very small values on the right, therefore having the appearance of being half-evolved compared to its counterpart. This probably indicates a low mutation rate, which is discussed now.




An important aspect of genetic algorithms is, that very few genes in the chromosome contribute to the phenotype, leading to the fact, that instead of continuous applied selective pressure everywhere, it is only applied sparely. The absence of selective pressure for some time on some genes means that these genes can be mutated negatively with no visible consequences. Therefore, a non-low mutation rate is sometimes problematic, because there may be ‘good’ evolutionary changes on some genes, which will most probably be corrupted while not expressed. On the other hand, the problem with a low mutation rate combined with long chromosomes leads to little exploration.

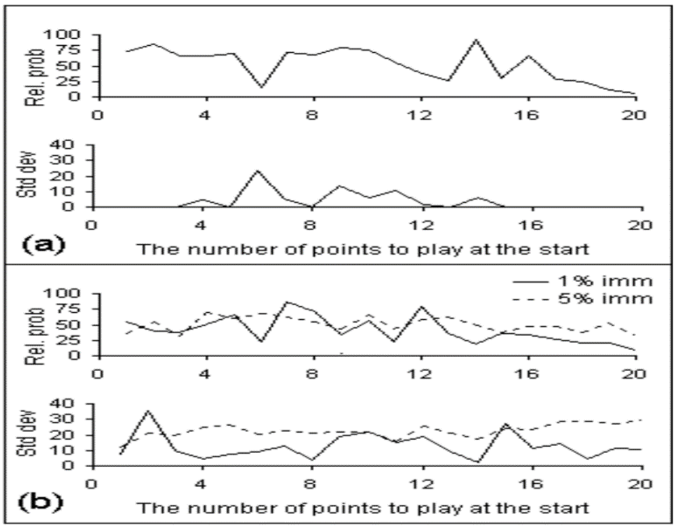
Hence a difficult trade-off arises:

$$\begin{gathered} safe\ building\ blocks\ \&\ insufficient\ exploration \\ vs \\ sufficient\ exploration\ \&\ unsafe\ blocks \end{gathered}$$

In the next Figure, you can see the outcome of an evolution with too much mutation. In 'a' it can be seen that though a desirable average genotype developed after 1000 generations, the standard deviations were quite high across the population, indicating a high variety. This shows, that even though there was a lot of mutation in this population, it did not lead to a precise genotype, because most likely 'good' evolutionary changes might be corrupted (sufficient exploration & unsafe blocks).

In part 'b' of the last figures documents the result of trying to bypass the problem of a non-low mutation rate whilst still guarantying a good exploration. Hoping that good genes would be able to enter the population, random immigrants were added in every generation.

The lower plot shows that in fact, because of the immigration there was an effective raising of the standard deviations/mutation rate. This characteristic developed because enough immigrants were able to 'survive' and pass their genes to the population via crossover.



# Conclusion

In this assignment the game Footsteps, as well as the evolving of strategies for this game, were discussed. It demonstrated that there is a lot more to the game than sheer luck and this can be applied to many other games and situations in everyday life. Moreover, it stated that the deterministic strategies are faulty, accounted by their exploitability. In fact, if you knew what the individual is going to do it will be easy to react to that turn.
In contrast it was found that the probabilistic strategies were noticeably better. But why is this? In a situation of attack or counterattack, as it is in our case of Footsteps, the best strategy is unpredictability, including chicanery and as well as randomness. As long as an opponent knows what you are going to do next the upcoming turn of yours will be exploitable and this is even if biased probabilities were used.
The example mentioned in the introduction about the game between Arsenal FC and Aston Villa prove this phenomenon quite accurate, because Villas Coach knew how Arsenal was going to play, he was able to exploit the opposites strategy. The only problem with this example is, that the coach did not only know what would be the best move in this situation, because he got to this state of information by analyzing what the opponent has done before. In conclusion the difference between the two approaches is, that Villas Coach based his knowledge on previous findings and the paper based their decision on the sheer state of the game. In my opinion Morris and Watson should also have included the decisions the opponent made in previous turns. For example, it would have been possible to find out what kind of player the individual is playing against, is it aggressive or passive. There where many aspects like this not discussed in the paper my assignment is based on. Maybe the research would be to sophisticated, if they would have taken more aspects in consideration.

Furthermore, it was concluded that in genetic algorithms it is important to measure convergence phenotypically not genotypically because only some of the chromosome's genes contribute to their individual's phenotypical development.

This may lead to a generation of the population, which is phenotypically converged to an acceptable solution, even though it is not converged genotypically. This case could give the impression that, from the genetic point of view, no solution had been found. As a consequence, using the wrong convergence metric includes wasting time, avoidable searching and also overlooking possible solutions to your problem.

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