Final exam Algorithmics SS 2019

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Hints:

Time limit: 120 minutes

Admitted appliances: none

Please give your answers and interim results exclusively in the pages of the assignments. If the space is not sufficient, you may use the blank reverse sheet on the opposite side.

Language: You may answer each assignment in German or English just as you feel most comfortable in order to express your thoughts and intentions clearly. In particular, you may also switch the language between or within the assignments.

This exam consists of 10 pages including this cover sheet.

This exam issues 50 evaluation credits (EC).
For passing this exam you need at least 25 EC.

Good luck!

Assignment 1: (8 EC)

a) Use mergesort to sort the sequence 7 8 3 2 4 1 6 5 in ascending order: In order to confine this task, just show the first and second recursive call, give the result of the second recursive call directly (without further details) and show the further actions in the first call until the result is achieved. (2 EC)

b) Do the same as in a) with quicksort: Give the Pivot element and the result after quicksort partition. You should confine the details again to 2 recursive steps For the Pivot element, you should always choose the best one for a minimum run time (other Pivot elements will cause a credit loss). Mention which was the criterion by which you determined the best Pivot element. (3 EC)

c) Give the recursive formula for the run time for mergesort and quicksort in the worst case. Mention the asymptotic complexity (no proof demanded). What is an advantage of quicksort with respect to mergesort? (3 EC)

Assignment 2: (7 EC)

a) For the recursive formula $T(n) \leq T(3/4\ n) + T\ (n/5) + cn$, prove that holds: $T(n) \in O(n)$. You must only show the inductive step from $<n$ to n. (4 EC)

b) For which algorithm is this formula relevant? Justify each of the 3 summands by the parts of this algorithm (just state what each part costs). (3 EC)

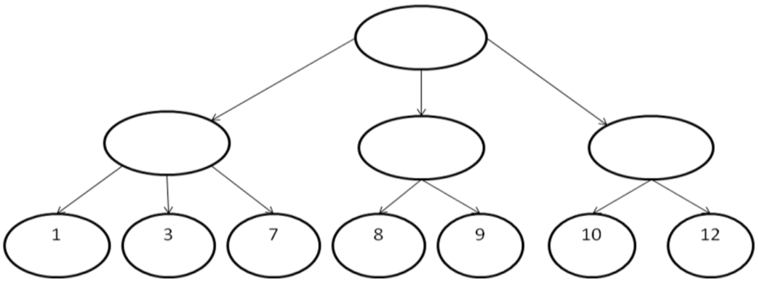
Assignment 3: (3 EC)

a) Specify the 3 operations belonging to the dictionary problem and tell the best asymptotic upper bounds in which they can be solved. You should only mention upper bounds that hold simultaneously for all operations. (2 EC)

b) Indicate in which cases you should rather choose a hashing procedure and in which cases a search tree procedure (no justification required). (1 EC)

Assignment 4: (5 EC)

a) Enter the keys into the internal nodes of the following (2,3) tree: (1 EC)



b) Modify this (2,3) tree by inserting the element 6. Update also the keys of the internal nodes. (2 EC)

c) In the tree obtained in b) delete the element 10. Update the keys of the internal nodes. (2 EC)

Assignment 5: (8 EC)

Consider the algorithm of Floyd-Warshall for computing the all pairs shortest paths:

for $i = 1, \ldots, n$ **do**
    for $j = 1, \ldots, n$ **do**
        $d_{ij}^{(0)} = \begin{cases} c(i,j): & \text{falls } (i,j) \in E \\ \infty: & \text{sonst} \end{cases}$
    end for
end for
for $k = 1, \ldots, n$ **do**
    for $i = 1, \ldots, n$ **do**
        for $j = 1, \ldots, n$ **do**
            $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$
        end for
    end for
end for

a) Specify the property of the coefficients $d_{ij}^{(k)}$. (1 EC)

b) Prove that the above property of $d_{ij}^{(k)}$ is always maintained by the algorithm: Sketch the complete proof via mathematical induction and make an illustration if this is helpful for your argumentation. (4 EC)

c) Determine the asymptotic run time and justify this by entering the times for the relevant substeps in the above algorithm. (3 EC)

Assignment 6: (4 EC)

State the theorem of Ford-Fulkerson and explain its relevance for the proof of correctness of the Edmonds-Karp algorithm.

Assignment 7: (5 EC)

Consider the following text T which should be investigated if and where it contains the pattern P:

index:	1	2	3	4	5	6	7	8	9	10	11	12
T:	a	b	a	c	b	a	a	b	b	a	b	c
P:	a	b	b	a								

a) Count how many comparisons the trivial method must perform until it has determined that position 7 is the first and only hit. Justify your number by showing exactly which comparisons must be explicitely performed. (2 EC)

b) Count how many comparisons the KMP algorithm must perform until it has determined that position 7 is the first and only hit. Justify your number by showing exactly which comparisons must be explicitely performed. (2 EC)

c) What is the run time improvement in general when T consists of n letters and P of k? (1 EC)

Assignment 8: (5 EC)

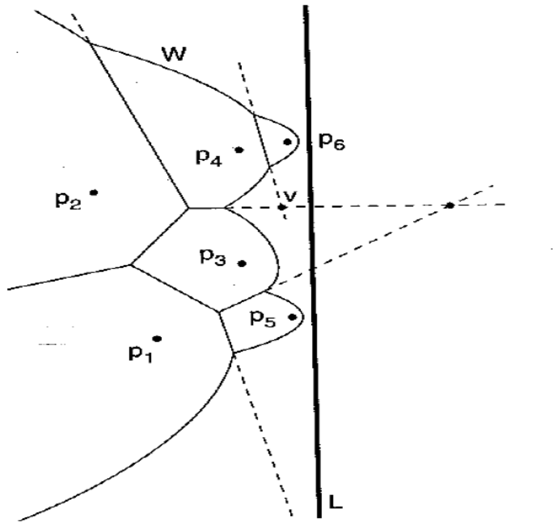
a) Sketch how the problem Closest Pair in the plane is solved in a trivial way. Specify and justify the asymptotic run time. (2 EC)

b) Sketch how Closest Pair in the plane may be solved when the Voronoi diagram is already computed. Specify and justify the asymptotic run time now. (2 EC)

c) Is it worth to compute the Voronoi diagram first when only the closest pair has to be computed? Justify your answer with the specification of run times. (1 EC)

Assignment 9: (5 EC)

Consider the following sweep line status in the course of the plane sweep computation of a Voronoi diagram:



a) In this status, what is the next event of the event point schedule? (1 EC)

b) When the event identified in a) is reached by L, how will the sweep status structure be changed? Tell exactly what data will vanish and what data will be inserted. (2 EC)

c) What does just the update of b) cost: Specify the asymptotic run time of this single step (not the entire algorithm). Which is the crucial property maintained for all objects of the sweep status structure in order to make this run time possible? (2 EC)