Final exam Algorithmics SS 2018

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Hints:

Time limit: 120 minutes

Admitted appliances: none

Please give your answers and interim results exclusively in the pages of the assignments. If the space is not sufficient, you may use the blank reverse sheet on the opposite side.

Language: You may answer each assignment in German or English just as you feel most comfortable in order to express your thoughts and intentions clearly. In particular, you may also switch the language between or within the assignments.

This exam consists of 11 pages including this cover sheet.

This exam issues 50 evaluation credits (EC).
For passing this exam you need at least 25 EC.

Good luck!

Assignment 1: (6 EC)

Analyse Quicksort:

a) Describe it in some form (words or pseudocode) (2 EC)

b) Give a recursive formula for its worst case run time.
Describe the situation which leads to the worst case. (1 EC)

c) Denote the complexity class of b) and give a sketch of a proof by mathematical induction by the following:
i) Denote what may be assumed and what should be shown in the proof.
ii) Show the first step of the proof using the inductive assumption. (3 EC)

If you finish the proof in c), you get 3 bonus credits.

Assignment 2: (6 EC)

Analyse binary search applied to an input array:

a) What is the prerequesite for the input in order to make binary search work correctly? (1 EC)

b) Give a recursive formula to describe the run time and denote its complexity class..(1 EC)

c) Prove the complexity class denoted in b) by mathematical induction. For simplicity, you may assume that the input size is a power of 2. (2 EC)

d) What is the benefit of the much more complicated algorithm quadratic binary search? Is there also a drawback? Denote the corresponding complexity classes in your answer. (2 EC)

Assignment 3: (5 EC)

Consider order statistics Select (k,A), i.e. the problem to select the kth element from an input array A:

a) What is the crucial difference in the functionality between the randomised and the deterministic algorithm? For this answer it is not necessary that you denote the differences in detail. (1 EC)

b) Denote the complexity classes for the **worst case** for both algorithms. (2 EC)

c) For the deterministic algorithm what is the key idea in order to achieve the optimum worst case behaviour. Denote the crucial constant and tell if it matters when that constant is decreased or increased (give answers for both directions). (2 EC)

Assignment 4: (5 EC)

Investigate Radixsort for sorting elements:

a) Demonstrate this procedure on the following input to be sorted lexicographically showing the intermediate results after each sorting step:

   Cesar, Lizzy, Anna, Doris, Berta, Bert, Chris (3 EC)

b) What is the run time of this procedure for the lexicographical sorting problem of names? Explain all parameters on which this procedure depends on. Also denote which parameter can be considered a constant and which is of variable size. (2 EC)

Assignment 5: (5 EC)

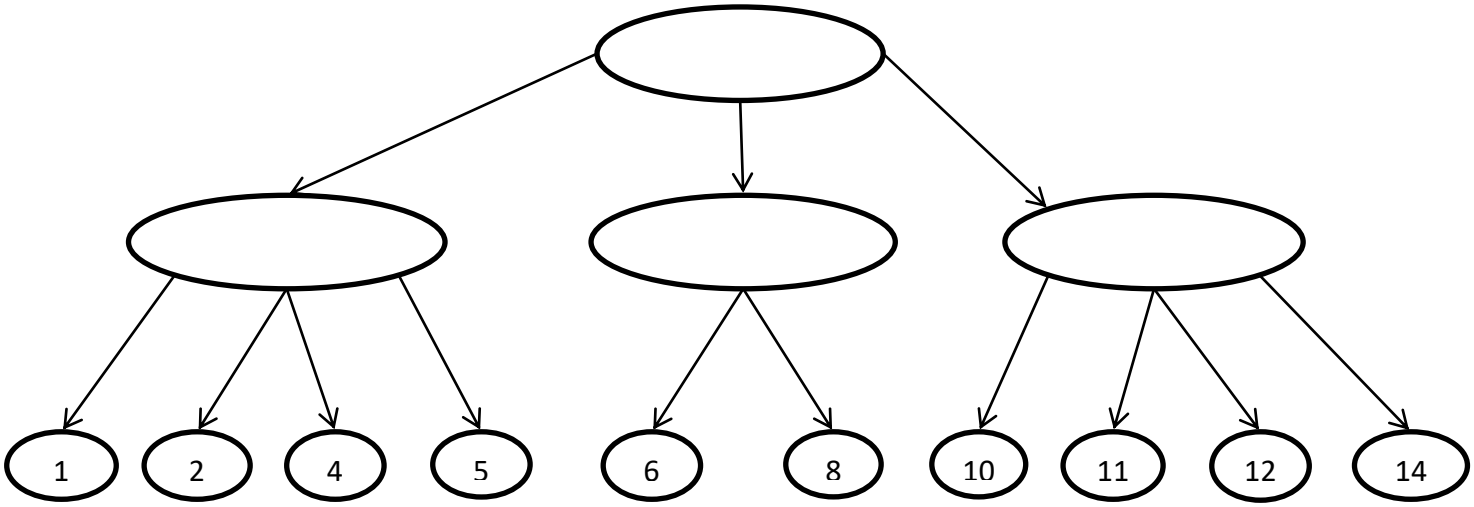
In the (2,4) tree below identify the data with the keys.

a) Attach the missing keys to all inner nodes of the tree. (1 EC)

For the following draw a new tree respectively:

b) Transform the tree into an equivalent red-black tree. The resulting red-black tree should have all data in the leaves only as well as the original one. (2 EC)

c) Insert the element 3 into the original tree using a systematic insert24 method. Update the keys. (2 EC)



Assignment 6: (3 EC)

Consider the following algorithm for computing an optimal binary search tree:

Algorithm 3: [Bellman, 1957] Iterative Suche nach dem optimalen Suchbaum T.

```
1:  for i = 0, ..., n do
2:      w_{i+1,i} = q_i
3:      c_{i+1,i} = 0
4:  end for
5:  for k = 0, ..., n − 1 do
6:      for i = 1, ..., n − k do
7:          j = i + k
8:          Determine m with  i ≤ m ≤ j,  s. that  c_{i,m−1} + c_{m+1,j}  is minimal .
9:          r_{i,j} = m
10:         w_{i,j} = w_{i,m−1} + w_{m+1,j} + p_m
11:         c_{i,j} = c_{i,m−1} + c_{m+1,j} + w_{i,j}
12:     end for
13: end for
```

a) Explain the meaning of the input values p_i and q_i and of the computed value $c_{i,j}$. (1 EC)

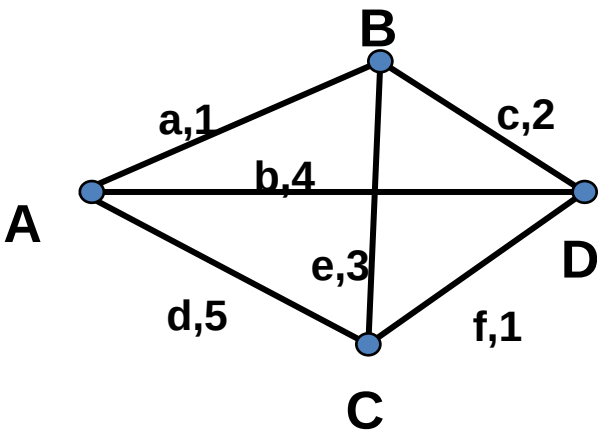
b) State and prove the asymptotic run time of this algorithm. The proof may be informal and refer to run time estimation of the lines above. (2 EC)

Assignment 7: (7 EC)

In the graph below the nodes are named by capital letters and the edges by small letters. The numbers at the edges denote their lengths.

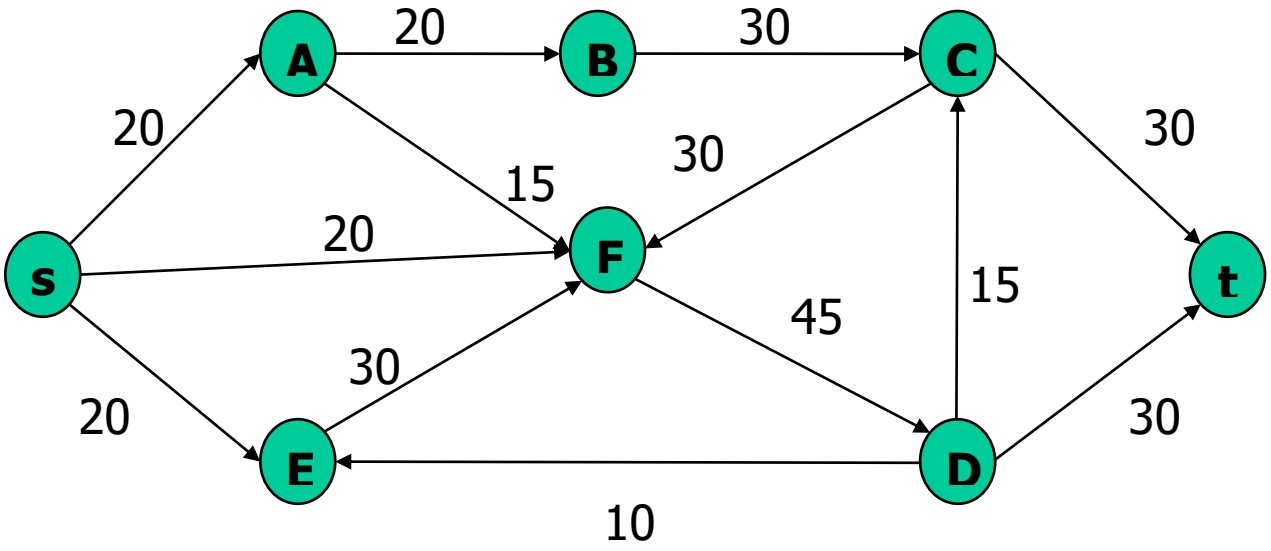
Initialise the algorithm of Kruskal inserting the edges in alphabetic order and show the resulting heap and the union-find structure.

Simulate the algorithm showing the new heap and the new union-find structure after each intake of an edge into the Kruskal forest. Your simulation should terminate only when the minimum spanning tree is computed.



Assignment 8: (5 EC)

Consider the following graph with the given flow capacities:



Simulate the first step of the algorithm of Edmonds-Karp by the following:

a) Find an augmenting path which Edmonds-Karp would choose. (2 EC)

b) Define a flow in the original graph obtained from this augmenting path. (1 EC)

c) Draw the corresponding residual graph. (2 EC)

Assignment 9: (5 EC)

Consider the following code for the prefix function of Knuth-Morris-Pratt:

```
1 π(1) := 0;
2 i := 2; q := 0;
3 while i ≤ m do
4 {
5     while (q>0) and (P[i]≠P[q+1]) do
6         q := π(q);
7     if P[i]=P[q+1] then q := q+1;
8     π(i) := q;
9     i := i+1;
10 }
```

For the following, let n be the length of the text and m be the length of the pattern to be matched in the text:

a) What is the general advantage of Knuth-Morris-Pratt compared to a straight forward method for string matching? (1 EC)

b) What is the run time of this function, and by which argument can this be proved? Refer specifically to this code and to some properties of the involved parameters/functions. (2 EC)

c) Apply the prefix function to the following pattern: (2 EC)

t o b e o r n o t t o b e

Assignment 10: (3 EC)

Consider the problem Convex Hull in 2 dimensions:

a) State the problem precisely: What is the required input and what is the desired output? (1 EC)

b) Describe how a Voronoi diagram can be used to get an efficient algorithm for Convex Hull. (2 EC)