Final exam Algorithmics SS 2016

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Hints:

Time limit: 120 minutes

Admitted appliances: none

Please give your answers and interim results exclusively in the space between the assignments. If the space is not sufficient, you may use the blank reverse sheet on the opposite side.

Language: You may answer each assignment in German or English just as you feel most comfortable in order to express your thoughts and intentions clearly. In particular, you may also switch the language between or within the assignments.

This exam consists of 10 pages including this cover sheet.

This exam issues 50 evaluation credits (EC).
For passing this exam you need at least 25 EC.

Good luck!

Assignment 1: (6 EC)

Analyse SelectionSort:

a) Describe it in some form (words or pseudocode) (2 EC)

b) Give a recursive formula for its worst case run time (1 EC)

c) Prove the best asymptotic class for the worst case run time using b) and mathematical induction. (3 EC)

Assignment 2: (2 EC)

Give a reason for practitioners why the focus on a better asymptotic run time class is more important than the focus on a better constant in searching for better algorithms for a given problem.

Assignment 3: (8 EC)

Apply the worst case optimal select algorithm for the following input:

Select (9, A) für A = 24 4 25 5 26 6 27 7 28 8 29 9 30 10 31

Choose the constant c=3 for the nonrecursive call.

a) Describe the first steps of the algorithm by giving the intermediate results. You may finish your description with the invocation of the first recursive call. (6 EC)

b) In your description, you have to split the input array in some way. Comment on the size of the split: May it be arbitrary? Which other sizes are you allowed to take without deteriorating the asymptotic worst case run time behavior? (2 EC)

Assignment 4: (7 EC)

Given n integers in decadic representation.

Consider the following sorting procedure:

Determine the maximum and its number of digits. Sort the numbers by radixsort.

a) Demonstrate this procedure on the following input showing the intermediate steps:
2002, 303, 4489, 1019, 448, 2020. (4 EC)

b) What is the run time of this procedure in general? (1 EC)

c) Comment how this compares to mergesort. (2 EC)

Assignment 5: (5 EC)

Consider the following algorithm for computing an optimal binary search tree:

Algorithm 3: [Bellman, 1957] Iterative Suche nach dem optimalen Suchbaum T.

```
for i = 0, ..., n do
    w_{i+1,i} = q_i
    c_{i+1,i} = 0
end for
for k = 0, ..., n-1 do
    for i = 1, ..., n-k do
        j = i + k
        Determine m where i ≤ m ≤ j, s. that c_{i,m-1} + c_{m+1,j} Is minimal
        r_{i,j} = m
        w_{i,j} = w_{i,m-1} + w_{m+1,j} + p_m
        c_{i,j} = c_{i,m-1} + c_{m+1,j} + w_{i,j}
    end for
end for
```

a) Explain the meaning of the input values p_i and q_i and of the computed value $c_{i,j}$.

b) State and prove the asymptotic run time of this algorithm. The proof may be informal and refer to run time estimation of the lines above.

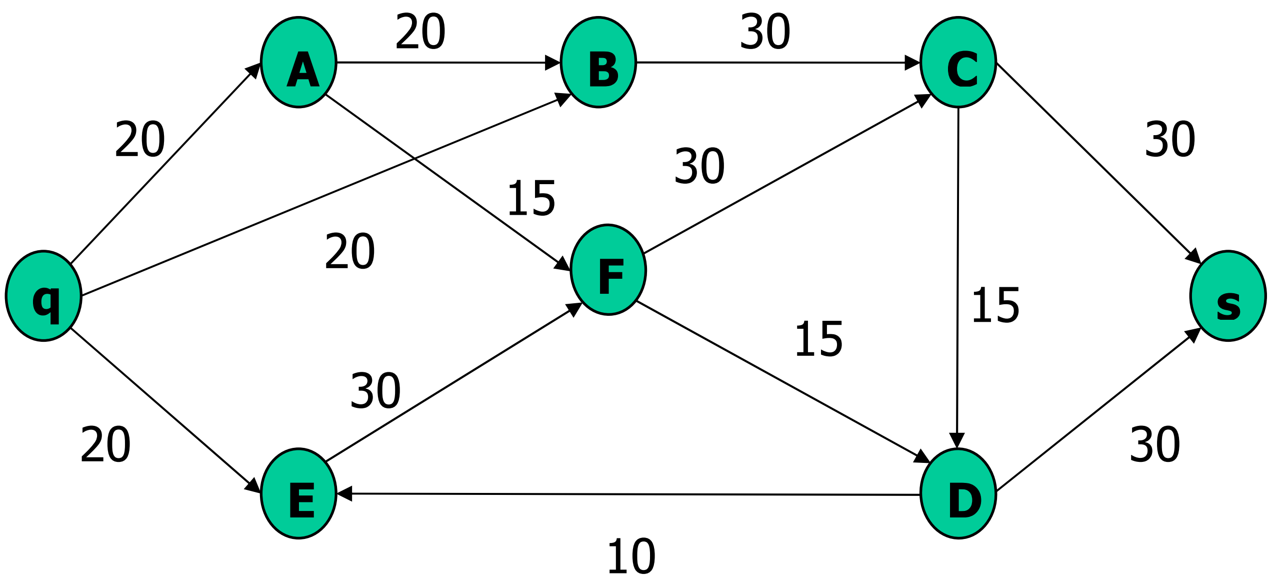
Assignment 6: (7 EC)

a) Describe the algorithm of Dijkstra for computing the shortest path between s and t in words (scheduled in separate steps) or in pseudocode. (2 EC)

b) Give a run time estimate for each step described in a) and give reasons by which subalgorithm resp. data structure you achieve the run time estimate for the respective step. Conclude in a run time estimate for the entire algorithm. (5 EC)

Assignment 7: (3 EC)

Consider the following graph with the given flow capacities:



Simulate the first step of the algorithm of Edmonds-Karp by the following:

a) Find an augmenting path which Edmonds-Karp would choose.

b) Define a flow in the original graph obtained from this augmenting path.

Assignment 8: (4 EC)

Consider the following code for the prefix function of Knuth-Morris-Pratt:

```
π(1) := 0;
i := 2; q := 0;
while i ≤ m do
{
    while (q>0) and (P[i]≠P[q+1]) do
        q := π(q);
    if P[i]=P[q+1] then q := q+1;
    π(i) := q;
    i := i+1;
}
```

a) What is the general advantage of Knuth-Morris-Pratt compared to a straight forward method for string matching?

b) What is the run time of this function, and by which argument can this be proved? Refer specifically to this code and to some properties of the involved parameters/funtions.

Assignment 9: (8 EC)

Consider the problem Convex Hull in 2 dimensions:

a) State the problem precisely: What is the required input and what is the desired output?

b) Sketch how the problem is solved in a trivial way and give a run time estimation. (4 EC)

c) Sketch how a Voronoi diagram can be used to get a better algorithm for Convex Hull. State the new run time estimation and give reasons for that. (3 EC)