

**Assignment 1)**

Given the following predicates on the set of all human beings:

$L(x,y)$ : x loves y

$F(x)$ : x is female

$M(x)$ : x is male

$C(x,y)$ : x ist child of y

Express the following statements by predicate logic combinations of only these four predicates. In particular, you are not allowed to confine the domains of definitions of the quantified variables or with additional functions.

- a) Erwin is the son of Hans.
- b) Hans is the father of Erwin.
- c) Linda loves all children of Hans.
- d) Linda loves only the children of Hans.
- e) Linda does not love children who are not of Hans, except if they are her own children.
- f) Every mother loves her children.
- g) Each person is either male or female.
- h) The love to a person is not always replicated.

**Assignment 2)**

Given the following predicates on the set of all human beings:

$L(x,y)$ : x loves y

$W(x,y)$ : x is married to y

$F(x)$ : x is female

$M(x)$ : x is male

Express the following statements by predicate logic combinations of only these four predicates. In particular, you are not allowed to confine the domains of definitions of the quantified variables or to work with additional functions.

- a) Anna is a woman while Bernd and Erwin are men.
- b) Anna loves Bernd, but is married to Erwin.
- c) Erwin loves all women.
- d) Only Anna loves Bernd.
- e) Bernd loves nobody except for himself.
- f) Erwin is loved by all women except for Anna.
- g) Only persons with different gender are allowed to marry.

- h) There is no person having both genders.
- i) The marriage to a person is a relation holding in both directions.
- j) Each human being has at most one spouse.
- k) Anna loves only such men who do not love other women.
- l) Erwin does not love men who love women he is also loving.

### Assignment 3)

Assume that all statements in assignment 2) are true and make a statement about the following questions. Answer: "True", "Not true" or "Cannot be answered". In the first two cases, also state which statements from assignment 2) are to be used as justification.

- a) Erwin loves Anna
- b) Bernd is married to Anna.
- c) Erwin loves Bernd.

### Assignment 4)

Given the following predicates:

- $\text{livesIn}(x,y)$  means that person  $x$  lives in place  $y$ .
- $\text{samePlace}(x,y)$  means that  $x$  and  $y$  live or are located in the same place (for persons and universities).
- $\text{studies}(x)$  means that person  $x$  is studies in a university.
- $\text{studiesAt}(x,y)$  means that person  $x$  is studies at university  $y$ .

Express the following statements by predicate logic combinations of only these four predicates. In particular, you are not allowed to confine the domains of definitions of the quantified variables or to work with additional predicates.

- a) Anna lives in Hamburg and studies at FH Wedel.
- b) Every university has enrolled students who live in the place of that university.
- c) If Otto studies, then only at a university in the city where he lives.
- d) Anna lives in the same place as Otto, but studies at a different university.

Does it follow from these facts that Hamburg has a university? Justify your answer.

**Assignment 5)**

Let  $S$  be the set of all students,  $M$  the set of all majors, and  $N = \{1, 2, 3, 4, 5\}$  the set of all exam grades (according to the German system).

The following predicate is given:

hasExamGrade  $(x,y,z)$  means that  $x$  has the exam grade  $z$  in major  $y$

Use this predicate to define the following new predicates and specify the domain of definition and image range:

- suitableAsTutor  $(x)$  means that  $x$  has a 2 or a 1 in DM.
- passesExam  $(x,y)$  means that  $x$  has received at least a 4 in major  $y$ .
- atLeastAsHard  $(x,y)$  means that all students have a grade in major  $x$  which is at most as good as in  $y$ .

**Assignment 6)**

Let  $S$  be the set of all students,  $C$  the set of all subjects, and  $N = \{1, 2, 3, 4, 5\}$  the set of all exam grades (according to the German system).

Given the following predicates:

- hasExamGrade  $(x,y,z)$  means that  $x$  has the exam grade  $z$  in subject  $y$
- passedExam  $(x,y)$  means that  $x$  passed the exam in subject  $y$ .
- hasChances  $(x)$  means that  $x$  passed some exam.
- atLeastAsHard  $(x,y)$  means that all students who fail subject  $y$  also fail subject  $x$ .

Express the following statements by predicate logic combinations of only these four predicates. In particular, you are not allowed to confine the domains of definitions of the quantified variables or to work with additional predicates.

- No one who fails the introductory course has got chances.
- Analysis is at least as hard as DM and PS1.
- Only students who passed the introductory course have got chances.
- Students who passed the introductory course also pass other exams.
- Nobody has grades in DM and PS1 which differ by more than 2.
- Karl failed Analysis, but has chances.

Are the 6 facts consistent in themselves, i.e. can they apply simultaneously?

- Erna has passed DM and Analysis, but unfortunately not PS1.

Are all 7 facts consistent in themselves?

**Assignment 7)**

Given the following predicates:

- studies (s,m,u): s studies the major m at university u
- hasAbiturGPA (s,n): s has the abitur grade n (n = 5 if s has no abitur at all)
- hasEntryGPA (s,n): s has the entry qualifying grade n (n = 5, if s has no entry qualification at all)

Express the following facts exclusively by a logical combination of these three predicates. You may also work with arithmetic comparison predicates.

- a) Everyone who has passed the Abitur has an entry qualification.
- b) All students of the FH Wedel have an abitur or an entry qualification.
- c) Only graduates with abitur GPA of at least 3.2 or entry qualification of at least 2.7 study at FH Wedel.
- d) All abitur graduates with GPA 1.0 study medicine or law.
- e) Anyone studying medicine or law has an abitur.
- f) Students having an abitur GPA of 1.0 and not studying medicine or law are studying computer science at FH Wedel.
- g) A student can only study one major (at the same time) at the FH Wedel.

**Assignment 8)**

Given the following predicates on the set of all human beings:

$L(x,y)$ : x loves y       $F(x)$ : x is female       $M(x)$ : x ist male

Express in natural speech what the following statements mean. Classify whether you consider the statement to be strong (difficult to fulfill) or weak (easy to fulfill).

- a)  $\forall x: M(x) \rightarrow L(x,x)$
- b)  $\forall x: M(x) \wedge L(x,x)$
- c)  $\forall x \forall y \exists z: (F(x) \wedge M(y) \wedge L(x,y)) \rightarrow (L(y,z) \wedge (z \neq y))$
- d)  $\forall x \forall y \exists z: (F(x) \wedge L(y,z) \wedge (z \neq y)) \rightarrow M(y) \wedge L(x,y)$

- e)  $\exists x: M(x) \rightarrow \neg \exists y: F(y) \wedge L(y,x)$
- f)  $\exists x: M(x) \wedge \neg \exists y: F(y) \wedge L(y,x)$
- g)  $\exists x \exists y: M(x) \wedge \neg F(y) \wedge L(y,x)$
- h)  $\exists x \forall y: M(x) \wedge (\neg F(y) \vee \neg L(y,x))$

**Assignment 9)**

Let  $m$  be from the set of human beings and  $x, y \in \mathbb{Z}$ .

Order the following conditions by strength / weakness:

- a)
  - i)  $m$  is studying
  - ii)  $m$  has got an entry qualification
  - iii)  $m$  is studying at FH Wedel
  - iv)  $m$  has got an abitur
  - v)  $m$  is studying in 5<sup>th</sup> semester at FH Wedel
  - vi)  $m$  is studying computer science
- b)  $x^2 > 0, x > 0, x > 10, x \geq 10, x < 0$

**Assignment 10)**

Let *Dates* be the set of all valid dates of the year. (E.g: 31.10.2013  $\in$  *Dates*)

Gegeben seien die folgenden Funktionen mit den zugehörigen Bedeutungen:

- Y (x):  $Dates \rightarrow \mathbb{N}$  outputs the year of date x (E.g.: Y(31.10.2013) = 2013)
- M (x):  $Dates \rightarrow \mathbb{N}$  outputs the month of date x (E.g.: M (31.10.2013) = 10)
- D (x):  $Dates \rightarrow \mathbb{N}$  outputs the day in the month of x (E.g.: D (31.10.2013) = 31)

Describe the following statements with a predicate logic expression, i.e. you may only use operations that are defined in the predicate logic. You may also use the functions and sets defined above.

- a) No year has a 30.02.
- b) A 31. is only available in the months January, March, May, July, August, October and December.
- c) Only years divisible by 4 may have a 29.02.