Name:

Discrete Mathematics WS 2017/2018

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Assignment sheet 01

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Assignment 1

Given the following phrases of natural speech. For each phrase do the following:

- Identify two propositions and assign a variable to each proposition.
- Assign each variable with a truth value.
- Determine the formula for the whole proposition.
- Determine the truth value of the formula.

	Phrase: propositon / variable / assignment	formula	truth value
a)	Angela Merkel is Germany's chancellor and empress of China. Example: Let proposition p be "Angela Merkel is Germany's chancellor", $p := \top$ Let proposition q be "Angela Merkel is empress of China", $q := \perp$	$p \wedge q$	$\perp$
b)	Angela Merkel is Germany's chancellor or empress of China.		
c)	Either Angela Merkel is Germany's chancellor or she is empress of China.		
d)	If I do not pass the final exam in DM, I will study at least 1 semester longer.		
e)	When there is low tide at the North Sea, one can walk to the islands.		
f)	It is day if and only if there is no night.		

Assignment 2

Translate the following propositions into phrases of natural speech.

Example:		$(p \to q)$
	translation	p: The adversary team has made more goals.
	key:	q: We have lost.
	Solution:	If the adversary team has made more goals, we have lost.

a) $(\neg p \to \neg q)$

p: n is divisible by 3.
q: n is divisible by 12.

b) $(\neg p \leftrightarrow q)$

p: It is day.
q: It is night.

c) $((p \vee \neg q) \to r)$

p:The car is broken.
q: The car has got some fuel in the tank.
r: The car must be towed.

Assignment 3

Consider the following propositions and tell which fact is necessary for the other and which is sufficient for the other:

a) Students who solved all exercise assignments will pass the final exam.

b) Who loves each other will tease each other.

c) Only idiots do no have an own will.

Assignment 4

Prove the following law's of propositional logic with truth tables:

a) Replacing the implication by $\neg$ and $\vee$

b) One of deMorgan's laws

c) One of the distribution laws

For the following assignments you may need the content of the lecture of October 19:

Assignment 5

Prove the logical principle of indirect proof either with a truth table or by the application of other logical laws.

Assignment 6

Prove by the successive application of equivalence rules that the following formulae are equivalent:

a) $(p \to q) \land (r \to q)$

b) $(p \lor r) \to q$

Assignment 7

a) Consider the propositional formula:

$$\forall x \in D\ \exists y \in D : y^2 = x$$

Plug in one of the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$ for D such that the propositional formula converts to a true proposition and plug in another of these sets for D such that thepropositional formula converts to a false proposition.

b) Try the same with the following propositional formulae:

$$\forall x \in D\ \exists y \in D : x^2 = y \quad \text{and} \quad \exists x \in D\ \forall y \in D : x^2 = y$$

Assignment 8

Consider the following predicates:

$H(x)$: x is happy.

$F(x)$: x is female.

$L(x, y)$: x loves y.

Transform each statement of the following phrases into formal propositions of predicate logic using exclusively the predicates above, the constants Anna and Bernd, and eventually some variables x or y for quantors:

a) Anna is happy.

b) Anna loves Bernd.

c) Bernd loves several women, and his love to Anna is replicated.

d) Anna is happy, if her beloved Bernd loves her.

e) Anna is only happy, if her beloved Bernd loves only her.